Section 4.3: Rules of Logarithms

Key Topics: product rule, quotient rule, power rule, change-of-base formula, half-life

RULES OF LOGARITHMS

Let M, N, and a be positive real numbers with $a \neq 1$ and let r be any real number.

Rule	Description	Examples
1. ***Product Rule:*** $\log_a (MN) =$ _____	The logarithm of the _____ of two (or more) numbers is the ____ of the logarithms of the numbers.	$\ln (5 \cdot 7) = \ln 5 + \ln 7$ $\log (3x) = \log 3 + \log x$ $\log_2 (5 \cdot 17) = \log_2 5 + \log_2 17$
2. ***Quotient Rule:*** $\log_a\left(\frac{M}{N}\right) =$ _____	The logarithm of the _____ of two numbers is the _____ of the logarithms of the numbers.	$\ln \frac{5}{7} = \ln 5 - \ln 7$ $\log \frac{5}{x} = \log 5 - \log x$
3. ***Power Rule:*** $\log_a M^r =$ _____	The logarithm of a number to the _____ r is r ____ the logarithm of the number.	$\ln 5^7 = 7 \ln 5$ $\log 5^{3/2} = \frac{3}{2} \log 5$ $\log_2 7^{-3} = -3 \log_2 7$

Write the expression $\log_7 \dfrac{x^3(3x+2)^2}{(x-5)^4}$ in expanded form.

Write the expression $2\ln(5x+3) - 5\ln(x-3) - 4\ln x$ in condensed form.

Compare the Rules of Exponents and Logarithms

Exponents	Logarithms
1. $a^x \cdot a^y =$ _____	$\log_a xy =$ _____
2. $\frac{a^x}{a^y} =$ _____	$\log_a \frac{x}{y} =$ _____
3. $(a^x)^y =$ _____	$\log_a x^y =$ _____

Digits and Common Logarithms

A _____ number K has _____ digits if and only if $\log K$ is in the interval _____.

CHANGE-OF-BASE FORMULA

Let a, b, and x be positive real numbers with $a \neq 1$ and $b \neq 1$. Then $\log_b x$ can be converted to a different base as follows:

$$\log_b x = \frac{\log_a x}{\log_a b} = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

_____ _____ _____

Use the base changing formula to compute $\log_3 12$.

Growth and Decay Formula

_____ **Form**	_____ **Form**
$A(t) =$ _____	$\ln\left(\frac{A(t)}{A_0}\right) =$ _____

HALF-LIFE FORMULA

The half-life h of a substance undergoing exponential decay at a rate k $(k < 0)$ is given by the formula

$$h = \_\_\_\_\_.$$