Section 8.2: Systems of Linear Equations in Three Variables

Key Topics: Gaussian elimination, nonsquare systems

Gaussian Elimination

OPERATIONS THAT PRODUCE EQUIVALENT SYSTEMS

1. __________ the position of any two equations.
2. _______ any equation by a _______ constant.
3. ____ a multiple of one equation to another.

List the seven steps of the Gaussian Elimination Method

Step 1 ____

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Step 2 ____

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Step 3 ____

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Step 4 ____

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Step 5 ____

Step 6 ____

Step 7 ____

Summary

To solve a system of three equations in three variables, x, y, and z.

Step 1 Eliminate ________ (say, x) from any____ equations (say, (1) and (2)).

Step 2 Eliminate the _____variable, x, from equations (1) and (3).

Step 3 The result of Steps 1 and 2 produces ____ equations in the variables y and z. Eliminate a _______variable (say, y) from these two equations. You will obtain an ___________________ , z. _________________ for z.

Step 4 Find the values of the remaining two variables, x and y, by__________. Write the solution as an ___________ .

Solve the system of linear equations $\begin{cases} 2x - y + 4z = -11 & (1) \\ x - 5y - 2z = -15 & (2) \\ -3x + 4y - 2z = 20 & (3) \end{cases}$.

Inconsistent System

If in the process of converting a linear system to triangular form an equation of the form_______occurs, where_______, then the system has ___ solution and is ___________.

Dependent Equations

If in the process of converting a linear system to _________ form

(i) an equation of the form $0 = k(k \neq 0)$_________occur but

(ii) an equation of the form ______ _____occur, then the system of equations has _________many solutions and the equations are _____________.

Solve each system.

$$\begin{cases} x - y + 3z = 10 \\ 2x + y - 2z = 10 \\ 2x - 2y + 6z = 20 \end{cases}$$

$$\begin{cases} x + 2y - z = 12 \\ 2x - y + z = 7 \\ 2x + 4y - 2z = 5 \end{cases}$$

Section 8.3: Partial-Fraction Decomposition

Key Topics: methods of fraction decomposition based on the form of the denominator $Q(x)$

Partial-Fraction Decomposition

OBJECTIVE

Find the partial-fraction decomposition of a rational expression.

Step 1 ______________________________

____

Step 2 ______________________________

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Step 3 ______________________________

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Step 4 ______________________________

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Step 5 ______________________________

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CASE 1: THE DENOMINATOR IS THE PRODUCT OF DISTINCT (NONREPEATED) LINEAR FACTORS

Suppose $Q(x)$ can be factored as

$$Q(x) = c(x - a_1)(x - a_2)\cdots(x - a_n),$$

with no factor repeated. The partial-fraction decomposition of $\frac{P(x)}{Q(x)}$ is of the form

$$\frac{P(x)}{Q(x)} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

where $A_1, A_2, \ldots, A_n$ are ________ to be determined.

Find the partial-fraction decomposition of $\dfrac{5x+7}{(x+3)(x-3)}$.

CASE 2: THE DENOMINATOR HAS A REPEATED LINEAR FACTOR

Let $(x - a)^m$ be the linear factor $(x - a)$ that is repeated m times in $Q(x)$.

Then the portion of the partial-fraction decomposition of $\dfrac{P(x)}{Q(x)}$ that corresponds to the factor $(x - a)^m$ is

____

where $A_1, A_2, \ldots, A_m$ are ________.

Find the partial-fraction decomposition of $\dfrac{x-2}{(x+1)^2(x-3)}$.

CASE 3: THE DENOMINATOR HAS A NONREPEATED IRREDUCIBLE QUADRATIC FACTOR

Suppose $ax^2 + bx + c$ is an irreducible quadratic factor of $Q(x)$. Then the portion of the partial-fraction decomposition of $\frac{P(x)}{Q(x)}$ that corresponds to $ax^2 + bx + c$ has the form:

____

CASE 4: THE DENOMINATOR HAS A REPEATED IRREDUCIBLE QUADRATIC FACTOR

Suppose the denominator $Q(x)$ has a factor $(ax^2 + bx + c)^m$, where $m \geq 2$ and $ax^2 + bx + c$ is irreducible. Then the portion of the partial-fraction decomposition of $\frac{P(x)}{Q(x)}$ that corresponds to the factor $(ax^2 + bx + c)^m$ has the form:

____

Find the partial decomposition of $\frac{x^2 - 2x}{(x^2 + 9)(x^2 + x + 7)}$

Section 8.4: Systems of Nonlinear Equations

Key Topics: system of nonlinear equations, solving systems of nonlinear equations using substitution and elimination

In systems of ___________ equations, ___________ equation is ___________.

Solve the system of equations $\begin{cases} x - y^2 = -3 & (1) \\ x - 4y = -7 & (2) \end{cases}$ by the Substitution Method.

Solve the same system by the Elimination Method.

Section 8.5: Systems of Inequalities

Key Topics: graphing a linear inequality, graphing systems of inequalities

Graphing a Linear Inequality in Two Variables

OBJECTIVE

Graph an inequality in two variables.

List the four steps of graphing an inequality in two variables.

Step 1 ______________________________

Step 2 ______________________________

____

Step 3 ______________________________

____

Step 4 ______________________________

____

Graph the inequality $2x-3y>12$.

Graph the solution set of the system of inequalities $\begin{cases} 3x+5y \geq 15 \\ 2y-x \leq 4 \end{cases}$.

To graph a nonlinear inequality, we follow the ___________ steps that we used in solving a ___________ inequality.

The procedure for solving a ___________ system of inequalities is ___________ to the procedure for solving a linear ___________.

Graph the solution set of the system of inequalities.

$$\begin{cases} y < x+4 \\ y \geq x^2+2 \end{cases}$$

Section 8.6: Linear Programming

Key Topics: linear programming, constraints, set of feasible solutions, objective function, optimal solution

Recall that if a function f with domain $[a, b]$ has a largest and a smallest value, the largest value is called the______________ and the smallest value is called the______________. The process of finding the maximum or minimum value of a quantity is called ____________.

The inequalities that determine the region S are called __________, the region S is called the _____________________, and $f = ax + by$ is called the ________________. A point in S at which f reaches its maximum (or minimum) value, together with the value of f at that point, is called an _______________.

Solution of a Linear Programming Problem

1. If a linear programming problem ___ a solution, it ____ occur at one of the _______ of the _____________________ .
2. A linear programming problem may have_____ solutions, but __________of them occurs at a______of the feasible solution set.
3. In any case, the______of the objective function is_______.

Solving a Linear Programming Problem

OBJECTIVE

Solve a linear programming problem.

List the six steps of solving a linear programming problem

Step 1 ____

____

Step 2 ____

Step 3 ____

Step 4 ____

Step 5 ____

Step 6 ____

Maximize $f = 27x + 15y$, subject to the constraints:
$x \geq 0$, $y \geq 0$, $2x + 3y \leq 12$, $x + 3y \leq 9$