Section 8.3: Partial-Fraction Decomposition

Key Topics: methods of fraction decomposition based on the form of the denominator $Q(x)$

Partial-Fraction Decomposition

OBJECTIVE

Find the partial-fraction decomposition of a rational expression.

Step 1 ____

____

Step 2 ____

____

Step 3 ____

____

Step 4 ____

____

Step 5 ____

____

CASE 1: THE DENOMINATOR IS THE PRODUCT OF DISTINCT (NONREPEATED) LINEAR FACTORS

Suppose $Q(x)$ can be factored as

$$Q(x) = c(x - a_1)(x - a_2)\cdots(x - a_n),$$

with no factor repeated. The partial-fraction decomposition of $\frac{P(x)}{Q(x)}$ is of the form

$$\frac{P(x)}{Q(x)} = \underline{\hspace{6cm}}$$

where $A_1, A_2, \ldots, A_n$ are ________ to be determined.

Find the partial-fraction decomposition of $\dfrac{5x+7}{(x+3)(x-3)}$.

CASE 2: THE DENOMINATOR HAS A REPEATED LINEAR FACTOR

Let $(x - a)^m$ be the linear factor $(x - a)$ that is repeated m times in $Q(x)$.

Then the portion of the partial-fraction decomposition of $\dfrac{P(x)}{Q(x)}$ that corresponds to the factor $(x - a)^m$ is

____

where $A_1, A_2, \ldots, A_m$ are ________.

Find the partial-fraction decomposition of $\dfrac{x-2}{(x+1)^2(x-3)}$.

CASE 3: THE DENOMINATOR HAS A NONREPEATED IRREDUCIBLE QUADRATIC FACTOR

Suppose $ax^2 + bx + c$ is an irreducible quadratic factor of $Q(x)$. Then the portion of the partial-fraction decomposition of $\frac{P(x)}{Q(x)}$ that corresponds to $ax^2 + bx + c$ has the form:

____

CASE 4: THE DENOMINATOR HAS A REPEATED IRREDUCIBLE QUADRATIC FACTOR

Suppose the denominator $Q(x)$ has a factor $(ax^2 + bx + c)^m$, where $m \geq 2$ and $ax^2 + bx + c$ is irreducible. Then the portion of the partial-fraction decomposition of $\frac{P(x)}{Q(x)}$ that corresponds to the factor $(ax^2 + bx + c)^m$ has the form:

____

Find the partial decomposition of $\frac{x^2 - 2x}{(x^2 + 9)(x^2 + x + 7)}$