Section 9.2: Matrix Algebra

Key Topics: equal matrices, matrix operations, properties of matrices

Equality of Matrices

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be _____, written $A = B$, if

1. A and B have the _____ order $m \times n$ (that is, A and B have the same number m of ____ and same number n of _______).
2. ___ $= b_{ij}$ for <u>all</u> i and j; that is, each (i, j)th entry of A is equal to the __________ (i, j)th entry of B.

Matrix Addition

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two ______ matrices, their **sum** $A + B$ is the $m \times n$ matrix defined by

$$A + B = \_\_\_\_\_\_\_\_\_\_,$$

for ___ i and j.

Scalar Multiplication

Let $A = [a_{ij}]$ be an $m \times n$ matrix and let c be a ____ number. Then the **scalar product** of A and c is denoted by cA and is defined by

$$cA = \_\_\_\_\_\_.$$

Matrix Subtraction

If A and B are two ______ matrices, then their **difference** is defined by

$$A - B = \_\_\_\_\_\_\_\_\_\_.$$

Subtraction $A - B$ is performed by subtracting the ____________ entries of B from those of A.

MATRIX ADDITION AND SCALAR MULTIPLICATION PROPERTIES

Let A, B, and C be $m \times n$ matrices and c and d be scalars.

1. $A + B =$ ____________________ Commutative property of addition

2. $A + (B + C) =$ ____________ Associative property of addition

3. $A + \mathbf{0} =$ ____________________ Additive identity property

4. $A + (-A) =$ ________________ Additive inverse property

5. $(cd)A =$ ____________________ Associative property of scalar multiplication

6. $1A =$ ________________________ Scalar identity property

7. $c(A + B) =$ _________________ Distributive property

8. $(c + d)A =$ _________________ Distributive property

RULE FOR DEFINING THE PRODUCT *AB*

In order to define the product AB of two matrices A and B, the number of _______ of A must be equal to the number of ____ of B. If A is an ______ matrix and B is a ______ matrix, then the product AB is an ______ matrix.

PRODUCT OF $1 \times n$ AND $n \times 1$ MATRICES

Suppose A is a ______ matrix and B is an ______ matrix:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \cdots a_n \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

We define the product AB by

$AB =$ ________________________.

Matrix Multiplication

Let $A = [a_{ij}]$ be an ______ matrix and $B = [b_{ij}]$ be a ______ matrix. Then the **product** AB is the ______ matrix $C = [c_{ij}]$, where the entry c_{ij} of C is obtained by matrix multiplying the ith ___ of A by the jth _______ of B.

The definition of the product AB says that

______________________________.

Properties of Matrix Multiplication

Let A, B, and C be matrices and let c be a scalar. Assume that each product and sum is defined. Then

1. $(AB)C =$ ________________ Associative property of multiplication

2. **(i)** $A(B + C) =$ _________ Distributive property

(ii) $(A + B)C =$ _________ Distributive property

3. $c(AB) =$ ________________ Associative property of scalar multiplication

If $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 0 \\ 6 & -7 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -4 & 0 \end{bmatrix}$, and $D = \begin{bmatrix} 2 & -4 \\ -1 & 5 \\ 8 & 0 \end{bmatrix}$, find each of the following, if possible:

(i) $A+B$ (ii) $3A-2B$ (iii) $A-D$ (iv) AC (v) AD