Section 9.3: The Matrix Inverse

Key Topics: inverse of a matrix, solving systems of linear equations using inverse matrices

The Inverse of a Matrix

Let A be an_____ matrix and let I be the $n \times n$ _____ matrix. If there is an $n \times n$ matrix B such that

$$AB = \_\_ \quad \text{and} \quad \_\_\_ = \text{I},$$

then B is called the **inverse** of A and we write $B =$ ___ (read "A inverse").

PROCEDURE FOR FINDING THE INVERSE OF A MATRIX

Let A be an $n \times n$ matrix.

1. Form the $n \times 2n$ augmented matrix $[A \mid I]$, where I is the _____ matrix.
2. _ there is a sequence of elementary row operations that transforms _____, then this same sequence of row operations will transform $[A \mid I]$ into $[I \mid B]$, where $B =$ ___.
3. _____ your work by showing that $AA^{-1} = I$.

If it is not possible to transform A into I by row operations, then A does not have an inverse. (This occurs if, at any step in the process, you obtain a matrix $[C \mid D]$ in which C has a row of zeros.)

Find the inverse (if it exists) of the matrix $A = \begin{bmatrix} 0 & 3 & 2 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$.

A Rule for Finding the Inverse of a 2 × 2 Matrix

The matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible if and only if $ad - bc \neq 0$. Moreover, if $ad - bc \neq 0$, then

$$A^{-1} = \_\_\_\_\_.$$

If $ad - bc = 0$, the matrix A does not have an inverse.

Find the inverse (if it exists) of the matrix $A = \begin{bmatrix} 5 & 3 \\ -1 & 2 \end{bmatrix}$.

Using the methodology in Example 6, solve the system of equations

$$\begin{cases} x - y + 2z = 12 & (1) \\ -2x + y + z = -11 & (2). \\ 3x + 2y + z = 11 & (3) \end{cases}$$