Section 9.4: Determinants and Cramer's Rule

Key Topics: determinant, minors, cofactors, Cramer's Rule

Determinant of a 2 × 2 Matrix

The **determinant** of a 2 × 2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

denoted by $\det(A)$, $|A|$, or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, is called the ________________ of A and is defined by

$$\det(A) = \_\_\_\_\_\_\_.$$

Evaluate the determinant $\begin{vmatrix} 3 & -1 \\ -5 & -2 \end{vmatrix}$.

Minors and Cofactors in an *n* × *n* Matrix

Let A be an $n \times n$ _____ matrix. The _____ M_{ij} of the element a_{ij} is the _______ ____ of the _________________ matrix obtained by _______ the ith row and the jth column of A. The __________ of the entry a_{ij} is defined by

$$A_{ij} = \_\_\_\_\_\_\_\_\_.$$

n by *n* Determinant

Let A be a _____ matrix of order _____. The **determinant** of A is the ____ of the entries in any ___ of A (or column of A) multiplied by their ________________.

Evaluate the determinant of $A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & 4 & 3 \\ -3 & -2 & 1 \end{bmatrix}$.

CRAMER'S RULE FOR SOLVING TWO EQUATIONS IN TWO VARIABLES

The system

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

of two equations in two variables has a unique solution (x, y) given by

$$x = \_\_\_ \quad \text{and} \quad y = \_\_\_$$

provided that D ___ 0, where

$$D = \_\_\_\_\_\_\_\_, \quad D_x = \_\_\_\_\_\_\_\_, \quad \text{and} \quad D_y = \_\_\_\_\_\_\_\_$$

Use Cramer's rule to solve the system $\begin{cases} -3x + 5y = 25 \\ 4x + 3y = -14 \end{cases}$.

CRAMER'S RULE FOR SOLVING THREE EQUATIONS IN THREE VARIABLES

The system

$$\begin{cases} a_1x + b_1y + c_1z = k_1 \\ a_2x + b_2y + c_2z = k_2 \\ a_3x + b_3y + c_3z = k_3 \end{cases}$$

of three equations in three variables has a unique solution (x, y, z) given by

$$x = \_\_\_, \quad y = \_\_\_, \quad \text{and} \quad z = \_\_\_$$

provided that $D \neq 0$, where

$$D = \_\_\_\_\_\_\_\_\_\_, \quad D_x = \_\_\_\_\_\_\_\_\_\_, \quad D_y = \_\_\_\_\_\_\_\_\_\_, \quad \text{and} \quad D_z = \_\_\_\_\_\_\_\_\_\_$$

Solve the system of equations $\begin{cases} x - y + 11z = 7 & (1) \\ y + 4z = -2 & (2) \\ x + y + 3z = 3 & (3) \end{cases}$.