

Section 10.4: The Hyperbola

Key Topics: equations of a hyperbola, foci, transverse axis, conjugate axis, vertices

Hyperbola

A **hyperbola** is the set of all points in the plane, the _____ of the _____ of whose distances from _____ points is _____. The fixed points are called the _____ of the hyperbola.

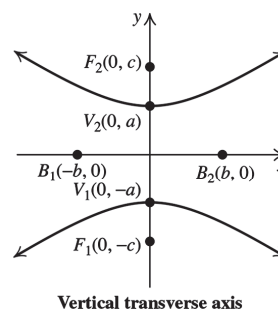
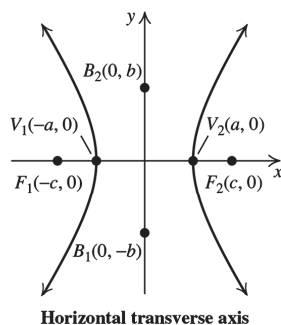
The two parts of the hyperbola are called _____.

SUMMARY OF MAIN FACTS

Main facts about hyperbolas centered at (0, 0)

| | $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1; a > 0, b > 0$ | $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1; a > 0, b > 0$ |
|-----------------------------|--|--|
| Standard Equation | $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1; a > 0, b > 0$ | $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1; a > 0, b > 0$ |
| Transverse axis on | _____ ($y = 0$) | _____ ($x = 0$) |
| Length of transverse axis | $2a$ | $2a$ |
| Conjugate axis on | y -axis _____ | x -axis _____ |
| Length of conjugate axis | $2b$ | $2b$ |
| Vertices | (____, 0) | (0, ____) |
| Endpoints of conjugate axis | (0, ____) | (____, 0) |
| Foci | (____, 0), where $c^2 =$ _____ | (0, ____), where $c^2 =$ _____ |
| Description | Hyperbola has a ____ branch and a ____ branch. (Hyperbola opens left and right.) | Hyperbola has an _____ and a ____ branch. (Hyperbola opens up and down.) |

Graph



Does the hyperbola $\frac{y^2}{15} - \frac{x^2}{20} = 1$ have its transverse axis on the x -axis or the y -axis? _____

Find the standard form of the equation of a hyperbola with vertices $(0, \pm 12)$ and foci $(0, \pm 13)$.

**THE ASYMPTOTES OF A HYPERBOLA WITH
CENTER $(0, 0)$**

1. The graph of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has transverse axis along the _____ and has the following two asymptotes:

_____ and _____

2. The graph of the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ has transverse axis along the _____ and has the following two asymptotes:

_____ and _____

Determine the asymptotes of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$.

Graphing a Hyperbola Centered at (0, 0)**OBJECTIVE**

Sketch the graph of a hyperbola in the form

(i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or (ii) $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Step 1 _____

Step 2 _____

Step 3 _____

Step 4 _____

Step 5 _____

Graph $25x^2 - 4y^2 = 100$.

SUMMARY OF **MAIN FACTS**

| Main properties of hyperbolas centered at (h, k) | | |
|--|--|--|
| Standard Equation | $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1;$ $a > 0, b > 0$ | $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1;$ $a > 0, b > 0$ |
| Transverse axis along the line | _____ | _____ |
| Length of transverse axis | $2a$ | $2a$ |
| Conjugate axis along the line | _____ | _____ |
| Length of conjugate axis | $2b$ | $2b$ |
| Center | (h, k) | (h, k) |
| Vertices | $(\text{____}, k)$ and $(\text{____}, k)$ | $(h, \text{____})$ and $(h, \text{____})$ |
| Endpoints of conjugate axis | $(h, \text{____})$ and $(h, \text{____})$ | $(\text{____}, k)$ and $(\text{____}, k)$ |
| Foci | $(\text{____}, k)$ and $(\text{____}, k);$ | $(h, \text{____})$ and $(h, \text{____});$ |
| Equation involving $a, b,$ and c | $c^2 = a^2 + b^2$ | $c^2 = a^2 + b^2$ |
| Asymptotes | $y - k = \text{____}(x - h)$ | $y - k = \text{____}(x - h)$ |

GENERAL EQUATION OF A HYPERBOLA

The graph of the equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is a hyperbola if _____. That is, _____ A nor C is zero, and they have _____ signs.