Chapter 11: Further Topics in Algebra

Section 11.1: Sequences and Series

Key Topics: sequences, recursive formula, Fibonacci sequence, $n!$, summation notation, summation properties, series

Sequence

An _______________ is a function whose domain is the set of positive integers. The function values, written as

____

are called the _____ of the sequence. The ***n*th term**, __, is called the _______ **term** of the sequence.

If the domain of a function consists of ____ the first _________ integers, the sequence is called a _____ **sequence**.

A ________ **formula** requires that one or more of the first few terms of the sequence ___ _______ and all other terms be defined __________ previously defined terms.

For example, the _________________ is a famous sequence that is defined _________ and shows up often ________. In this sequence, we ______ the _______ terms as $a_0 = 1$ and $a_1 = 1$; each subsequent term is the ____ of the two terms __________ preceding it. So we have

$a_0 = 1, a_1 = 1, a_2 = a_0 + a_1 = 1 + 1 = 2$ Add the two given terms.

$a_1 = 1, a_2 = 2, a_3 = a_1 + a_2 = 1 + 2 = 3$ Add the two previous terms.

$a_2 = 2, a_3 = 3, a_4 = a_2 + a_3 = 2 + 3 = 5$ Add the two previous terms.

$a_3 = 3, a_4 = 5, a_5 = a_3 + a_4 = 3 + 5 = 8$ Add the two previous terms.

The first six terms of the sequence are then

____

This sequence can also be defined in subscript notation:

____

Write the fourth term of the recursively defined sequence $a_1 = -5,\ a_{n+1} = 6 - 3a_n$.

Factorial

For any ____________ n, ***n factorial*** (written as $n!$) is defined as

$$n! = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

As a ______ case, ***zero factorial*** $(0!)$ is defined by

____

Write the sixth term of the sequence whose general term is $a_n = \dfrac{(-1)^{n-1}(11)^n}{(n+1)!}$.

SUMMATION NOTATION

The sum of the first $n\,(\geq 1)$ terms of a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is denoted by

$$\sum_{i=1}^{n} a_i = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

The letter i in the summation notation is called the ________________, n is called the __________, and 1 is called the __________ of the summation.

Summation Properties

Let a_k and b_k represent the general terms of two ________ and let c represent any __________. Then

1. $\sum_{k=1}^{n} c =$ ____

2. $\sum_{k=1}^{n} ca_k =$ _______

3. $\sum_{k=1}^{n} (a_k + b_k) =$ _____________

4. $\sum_{k=1}^{n} (a_k - b_k) =$ _____________

5. $\sum_{k=1}^{n} a_k =$ ____________________________

Series

Let $a_1, a_2, a_3, \ldots, a_k, \ldots$ be an ______ sequence. Then

1. The ____ of __ terms of the sequence is called a _____ and is denoted by

$$a_1 + a_2 + a_3 + \cdots = \_\_\_\_\_$$

2. The ____ of the _____ terms of the sequence

$$a_1 + a_2 + a_3 + \cdots + a_n = \_\_\_\_\_$$

is called the ______________ of the series.

Write the sum $8+11+14+\cdots+59+62+65$ in summation notation.