Section 11.3: Geometric Sequences and Series

Key Topics: geometric sequence definitions, general term, sum of the terms of geometric sequences, annuity value

Geometric Sequence

The sequence

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

is a ________________, or a **geometric progression**, if there is a number r such that each term ____________ in the sequence is obtained by ________________________ ____. The number r is called the ____________ of the geometric sequence. We have

$$\_\_\_\_\_\_ = r, \quad n \geq 1$$

Recursive Definition of a Geometric Sequence

A geometric sequence $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ can be defined recursively. The recursive formula

$$a_{n+1} = \_\_\_, \quad \_\_\_\_\_\_$$

defines a geometric sequence with the __________ and the _____________.

THE GENERAL TERM OF A GEOMETRIC SEQUENCE

Every geometric sequence can be written in the form

$$a_1, a_1r, a_1r^2, a_1r^3, \ldots, a_1r^{n-1}, \ldots,$$

where r is the common ratio. Since $a_1 = a_1(1) = a_1r^0$, the ________ **of the geometric sequence** is

$$a_n = \_\_\_\_\_\_, \text{ for } \_\_\_\_\_\_.$$

For the geometric sequence $2, 6, 18, 54, \ldots$ find a_n.

Further Properties of a Geometric Sequence (with ratio $r > 0$)

1. The general term of a geometric sequence can be obtained from the __________ (type) function $f(x) = A \cdot B^x$. That is,

$$a_n = f(n) = \_\_\_\_\_, \qquad \text{where } A = \_\_ \text{ and } B = \_\_ .$$

2. Points on the graph of the geometric sequence lie on the graph of the equation $y =$_______ .
3. The geometric sequence exhibits the following growth patterns:
   - If $B = r$ ___1 the geometric sequence represents **exponential** ________ .
   - If 0___ $B = r$ ___1 the geometric sequence represents **exponential** ______.

SUM OF THE TERMS OF A FINITE GEOMETRIC SEQUENCE

Let $a_1, a_2, a_3, \ldots, a_n$ be the first n terms of a geometric sequence with first term a_1 and common ratio r. The ______ of these terms is

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_ .$$

Find the sum $\sum_{i=1}^{10} 4(1.1)^i$, rounded to two decimal places.

VALUE OF AN ANNUITY

Let __ represent the ________ in dollars made at the end of each of ________________ ______ per year and let _ be the ________________. Then the value A of the annuity after t years is:

$$A = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

SUM OF THE TERMS OF AN INFINITE GEOMETRIC SEQUENCE

If $|r| < 1$, the infinite sum

$$a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1} + \cdots$$

is given by

$$S = \sum_{i=1}^{\infty} a_1r^{i-1} = \_\_\_\_\_\_\_$$

Find the sum $6 + 4 + \frac{8}{3} + \ldots$.