

## Section 3: The Square of Opposition

Any two propositions that have exactly the same subject and exactly the same predicate will have some logical relation to each other. However the two, propositions must match exactly with respect to both subject and predicate. For example, there may be no interesting logical relation between ‘All swans are parakeets’, and ‘Most non-swans are not parakeets’, since these two propositions do not have exactly the same subject. But there will be a logical relation between ‘All swans are non-parakeets’ and ‘Most swans are non-parakeets’. A **Diagram of Opposition** is a diagram of the logical relations between statements with exactly matching terms. Since classical syllogistic logic used only four types of categorical propositions, the diagram formed a square, called the **Square of Opposition**. In this system, adding the three intermediate quantifiers makes the square into more of an oblong—but we will continue to call the diagram a Square of Opposition.

This section explains the relations on the Square of Opposition and shows that most of the relations may be derived from a basic few. This chapter also includes a discussion of existence claims in syllogistic logic. The final group of exercises show students how to combine the relations on the Square of Opposition with immediate inferences to produce relatively complex chains of inference.

### The Four Types of Relations

In this system, any two propositions that have exactly the same subject and exactly the same predicate will have one of the four following logical relations to each other: contrary, subcontrary, contradictory, or implication.<sup>1</sup> Let’s consider these four relations one at a time.

#### Contrary

Contrary statements are opposed to each other, but they are not direct opposites. They are opposed to each other because they cannot both be true. If one is true, the other must be false. However, they are not direct opposites, since both of them could be false.

**contrary** - both cannot be true (but both could be false).

Suppose that two propositions are contrary to each other, and we know that the first of them is true. It follows that the second one must be false, since they cannot both be true. On the other hand, suppose we know only that the first one is false. In that case we cannot draw any inference at all about the second one. It is consistent to imagine that the second one might be either true or false.

*Examples:*

<p>‘All sharks are predators’ is contrary to ‘No sharks are predators’, so if ‘All sharks are predators’ is true, ‘No sharks are predators’ is false.</p>
<p>‘Most sharks are predators’ is contrary to ‘Most sharks are not predators’, so if ‘Most sharks are not predators’ is true, ‘Most sharks are predators’ is false.</p>
<p>‘Many sharks are predators’ is contrary to ‘No sharks are predators’, so if ‘Many sharks are predators’ is false, ‘No sharks are predators’ remains undetermined.</p>

### **Sub-contrary**

The sub-contrary relation exactly mirrors the contrary relation. Sub-contrary statements are not opposed to each other: both could be true. On the other hand, one or the other of them must be true. If we know that one of them is false, then we know that the other one must be true, since they cannot both be false.

**sub-contrary** - both cannot be false (but both could be true).

Suppose that two propositions are sub-contrary to each other, and we know that the first of them is true. In that case we cannot yet draw an inference about the second one. It is consistent to imagine that the second one might be either true or false. But suppose we know that the first one is false. Then we can infer that the second one is true.

*Examples:*

<p>‘Some sharks are predators’ is sub-contrary to ‘Some sharks are not predators’, so if ‘Some sharks are predators’ is true, ‘Some sharks are not predators’ remains undetermined.</p>
<p>‘Many sharks are predators’ is sub-contrary to ‘Many sharks are not predators’, so if ‘Many sharks are not predators’ is true, ‘Many sharks are predators’ remains undetermined.</p>
<p>‘Almost all sharks are predators’ is sub-contrary to ‘Some sharks are not predators’, so if ‘Some sharks are not predators’ is false, then ‘Almost all sharks are predators’ is true.</p>

### **Contradictory**

Contradictory statements are direct opposites. Whatever truth-value one of them has, the other must have the opposite. One way to think of the contradictory relations is as a combination of the contrary and sub-contrary relations. Like contrary statements, contradictories cannot both be true. If one is true, the other must be false. But like sub-contrary statements, contradictories cannot both be false. If one is false, the other must be true.

**contradictory** - both cannot be true, and both cannot be false.

In the case of contradictory statements, we can always draw an inference from one of the two statements to the other. The truth-value of one statement always fully determines the truth-value of its contradictory.

*Examples:*

'Some sharks are predators' is contradictory to 'No sharks are predators', so if 'Some sharks are predators' is true, 'No sharks are predators' is false.
'Many sharks are predators' is contradictory to 'Few sharks are predators', so if 'Few sharks are predators' is false, 'Many sharks are predators' is true.
'All sharks are predators' is contradictory to 'Some sharks are not predators', so if 'Some sharks are not predators' is true, 'All sharks are predators' is false.

### **Implication**

The implication relation is the most difficult to explain, perhaps because the implication relation is the one most central to the study of logic. The implication relation is the relation that premisses have to conclusions. In certain respects the whole study of logic is an attempt to understand the implication relation.

Suppose that we have two propositions, A and B. To say that A implies B is to say that the truth of B follows from the truth of A. A cannot be true unless B is also true. Thus, if we know that A is true, we also know that B is true. This relation also has a flip-side. Since A cannot be true unless B is true, B cannot be false unless A is also false. Thus, if we know that B is false, we also know that A is false.

**implication** - the statements may be ordered such that one cannot be true unless the other is also true.

Like contrariety and sub-contrariety, the implication relation leaves some things undetermined. Suppose again that A implies B. If we know that B is true, we cannot draw any inference about A. It is consistent to imagine that A might be either true or false. Similarly, if we know that A is false, we cannot draw any inference about B. It is consistent to imagine that B might be either true or false.

*Examples:*

‘Most sharks are predators’ implies ‘Some sharks are predators’, so if ‘Most sharks are predators’ is true, ‘Some sharks are predators’ is true.

‘Few sharks are predators’ implies ‘Most sharks are not predators’, so if ‘Most sharks are not predators’ is false, ‘Few sharks are predators’ is false.

‘All sharks are predators’ implies ‘Almost all sharks are predators’, so if ‘All sharks are predators’ is false, ‘Almost all sharks are predators’ remains undetermined.

### **The Problem of Existence**

Lewis Carroll, the author of *Alice in Wonderland*, wrote children’s books only as a hobby. By profession he was a mathematician and logician. There is a joke told about Lewis Carroll that one year he set about to organize a Logicians’ Club. Anyone was eligible to join, but only on the condition that he be willing to conduct himself at all times in an entirely logical manner. One day Lewis Carroll ran into a friend of his.

“How’s your Logic Club going, Charles?” the friend asked. (Lewis Carroll’s real name was Charles Lutwidge Dodgson.)

“Fine,” Lewis Carroll replied. “Every one of the members has solemnly agreed to the club rule, to conduct himself at all times in a logical manner.”

“Wonderful!” his friend expostulated. “How many members do you have?”

“Why, none,” was the reply. “But they have all agreed to the rule.”

Logicians of the 19th century, including especially George Boole, did not consider a statement of the form ‘All S are P’, to require the existence of members in either of the two classes, S and P, in order to be true. Indeed a statement in which the subject term had no members was considered to be true by default. These logicians would have said that the statement, ‘All members of Lewis Carroll’s Logic Club are humans with green and pink polka dot skin’, was true, precisely because there *were no* members of Lewis Carroll’s Logic Club. Hence *every one of them* had green and pink polka dot skin. By contrast, the same logicians thought that a statement of the form ‘Some S are P’, *did* require members in the classes S and P in order to be true. They would have said that the statement ‘Some members of Lewis Carroll’s Logic Club are logicians’, was false, again precisely because there were no members of Lewis Carroll’s Logic Club.

Normally (given the minimal interpretation of ‘some’) we would expect that if, ‘All sharks are predators’, is true, then ‘Some sharks are predators’, would also be true. Logicians rejected this view because the newly developed symbolic logic was incapable of showing the validity of this inference.<sup>2</sup> Since the contrary and sub-contrary relations on the Square of Opposition are derived from the implication relations (as will be shown presently), the symbolic logic was also incapable of showing the validity of contraries and sub-contraries. Only the contradictory relations on the Square of Opposition could be accepted as valid.

If we suppose that Particular statements make an implied existence claim, while Universal statements do not, then the inference from ‘all’ to ‘some’ should be invalid. Consider

the following chain of reasoning. Assume that the inference from ‘all’ to ‘some’ is valid. If Universal statements do not make an implied existence claim, then the statement ‘All members of Lewis Carroll’s Logic Club are humans with green and pink polka dot skin’, is true. It follows that ‘Some members of Lewis Carroll’s Logic Club are humans with green and pink polka dot skin’, is also true. But if Particular statements do make an implied existence claim, then we have apparently reasoned from a true premiss to the conclusion that there are humans with green and pink polka dot skin. Naturally this is absurd. Hence the inference from ‘all’ to ‘some’ must be invalid.

However, in most instances, the inference from ‘all’ to ‘some’ does not *feel* invalid. Moreover, we normally think it is possible to assert false Universal propositions about non-existent objects, and true Particular propositions. For example, it seems to me to be clear that ‘All unicorns are two-horned animals’, is false, since any creature that had more than one horn would, by definition, fail to be a unicorn. It seems to me to be equally clear that ‘Some unicorns are males’ and ‘Some unicorns are females’, are both true, since unicorns are thought of as creatures that reproduce sexually. Hence some *portion* of the class (which is not the same as saying some *member* of the class) must be male while the complementary portion must be female. This is not to say that some male unicorn *exists*. It is just to say that the class ‘unicorn’, in order to be the class of beings that we conceive it to be, must be capable of being partitioned along sexual lines. If the class is not divisible along these lines, then it is not unicorns that we are talking about, but some other type of creature.

Hence I feel that the inference from ‘all’ to ‘some’ should be regarded as valid. The truth or falsity of propositions, either Universal or Particular, does not reside solely in whether objects exist or not. To some extent the connection between subject and predicate must depend upon the meanings of the terms; and clearly any genuinely meaningful connection that obtains universally will obtain in (possibly hypothetical) particular cases as well.

If we accept this argument, then we may reject the claim that Particular statements make an implied existence claim while Universal statements do not. Every categorical proposition may be understood as making precisely the same type of existence claim as every other categorical proposition. But this may still be understood to mean two different things.

- (1) All categorical propositions make an implied existence claim.
- (2) No categorical propositions make an implied existence claim.

Aristotle is usually understood to have taken the first of these two options. This view has certain advantages. For one thing, it allows us to regard the peculiar claim that all the members of Lewis Carroll’s Logic Club have green and pink polka dot skin as simply false. No wonder the claim sounds peculiar! But on the negative side, the claim that all unicorns are one-horned would also be false. Indeed it would be impossible to say anything true about unicorns at all, except perhaps, ‘Unicorns don’t exist’.

Perhaps this difficulty can be overcome by introducing the notion of a ‘universe of discourse’. Perhaps we could say that every categorical proposition makes an implied existence claim, but only about the ‘universe’ that we are talking about. Thus the claim, ‘All unicorns are



Let us take a closer look at the relations represented on this Square, just to be sure they are correct. There are two assumptions concerning the interpretation of the propositions that must be kept in mind. Both assumptions have already been discussed, but they are important enough to bear repeating at this point.

Assumption #1. Each of the quantifiers takes either minimal or maximal interpretation. ‘Most’, ‘many’, and ‘some’ each state a minimum quantity, but a proposition beginning with one of these quantifiers is not false when that quantity is exceeded. ‘Few’ and ‘no’, on the other hand, state a maximum quantity, but a proposition beginning with ‘few’ or ‘no’ is not false if that quantity is not attained. Because of the way ‘all’ and ‘almost all’ are defined, they also must be understood as making only a minimal claim.

Assumption #2. Each of the quantifiers makes *no* existence claim.

### **Implication Relations**

Given the two stated assumptions, P statements follow from A statements by implication. Remember that ‘almost all’ merely states a minimum, but continues to be true for more than that quantity. Since ‘all sharks’ is more than ‘almost all sharks’, it follows that if ‘All sharks are predators’ is true, then ‘Almost all sharks are predators’ must also be true.

A similar argument can be given to show that if ‘Almost all sharks are predators’ is true, then ‘Most sharks are predators’ must also be true. Since ‘most’ means ‘more than half’, it should be obvious that ‘almost all sharks’ is more than merely ‘most sharks’. Hence T statements follow from P statements by implication.

It is not so obvious that K statements follow from P statements. Consider a group of three objects, say three sonnets of which two are Petrarchan (which indicates a particular rhyme scheme) while the third is Shakespearean. It is certainly true to say ‘Most of the Sonnets are Petrarchan sonnets’, since more than half of them are Petrarchan sonnets. But is it also true to say ‘Many of the Sonnets are Petrarchan sonnets’? Two sonnets hardly seems like ‘many’. But I think it is true, since we are not, after all, considering every sonnet ever written. We are considering only these three sonnets. Two out of three is certainly a significant proportion, and can, therefore, be characterized as ‘many’, even though we are not considering a very large total number. Hence K statements do follow from T statements by implication.

Finally, it should be obvious that I statements follow from K statements by implication. If ‘Many sharks are predators’ is true, then ‘Some sharks are predators’ must also be true.

The same arguments can be given to justify the implication relations on the negative side of the Square. Since ‘none’ is even less than ‘few’, if ‘No sparrows are parakeets’ is true, then ‘Few sparrows are parakeets’ must also be true. Of course ‘Few sparrows are parakeets’ is a bit of an understatement, but the maximal interpretation of ‘few’ requires that such understatements be allowed as true. Hence B statements follow from E statements by implication.

Similarly D statements follow from B statements by implication. ‘Few’ takes maximal interpretation, while ‘most’ takes minimal interpretation; but it should nevertheless be clear that if ‘Few sparrows are parakeets’ is true, then ‘Most sparrows are not parakeets’ must also be true.

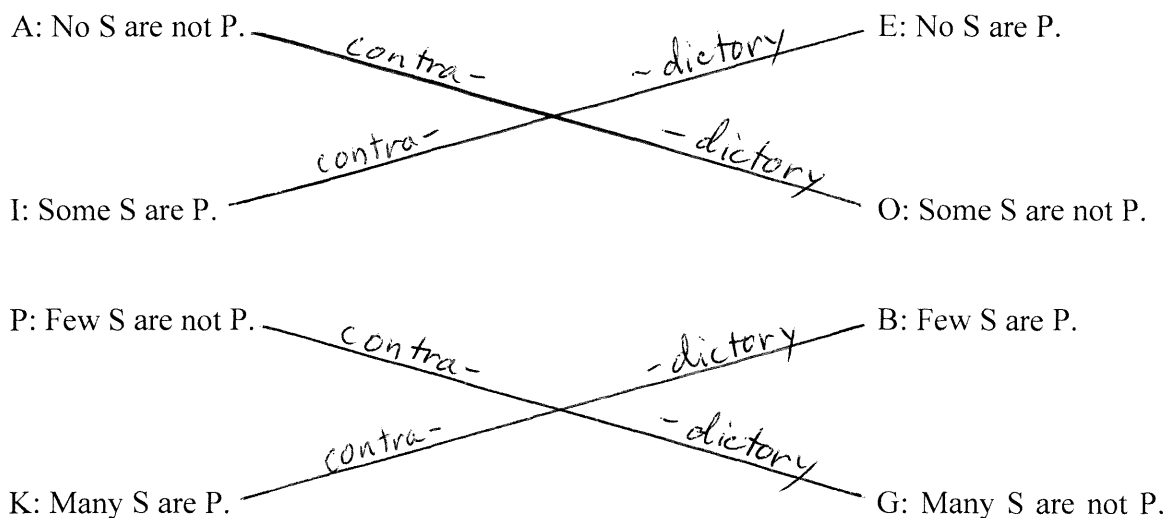
G statements follow from D statements by implication. If ‘Many sparrows are not parakeets’ is true, then ‘Many sparrows are not parakeets’ is also true; though again it may be important to remember that both ‘most’ and ‘many’ mean only most and many of the class we are talking about.

Finally, at the bottom of the square, O statements follow from G statements by implication. If ‘Many sparrows are not parakeets’ is true, then ‘Some sparrows are not parakeets’ is also true.

Notice that the implication relation is a transitive relation. If A implies B and B implies C, it follows that A implies C. Thus, since A statements imply P statements, and P statements imply T statements, it follows that A statements imply T statements. Since T statements imply K statements, it further follows that A (and P) statements imply K statements. Finally, of course, A (as well as P and T) statements imply I statements. If ‘All sharks are predators’ is true, then ‘Some sharks are predators’ must also be true.

### Contradictory Relations

There are four contradictory relations on the basic Square of Opposition. All four are most easily explained if we state the categorical propositions on the Square in their unabbreviated form. Let us separate the big Square into the two smaller squares that make it up.



Pairs of words that have the opposite meaning are known as ‘antonyms’. Replacing a word with its antonym should have the effect of altering the meaning of the sentence—indeed it might change the meaning so much that the sentence is made to express exactly the opposite meaning. That seems to be precisely what is happening on the Square of Opposition.

The words ‘some’ and ‘none’ are antonyms. The proposition ‘No sparrows are parakeets’ has precisely the opposite meaning of the proposition ‘Some sparrows are parakeets’. If either one of these propositions is true, the other must be false; and if either one is false, the other must be true. The same can be said when we add a negative particle. The proposition ‘No sharks are not predators’ has precisely the opposite meaning of the proposition ‘Some sharks are not



predators'. If either of these propositions is true, the other must be false; and if either is false, the other must be true.

The words 'few' and 'many' are also antonyms. 'Many' means 'a significant proportion', while 'few' means '*not* a significant proportion'. Thus again, the proposition 'Few sparrows are parakeets' has precisely the opposite meaning of 'Many sparrows are parakeets'. If either of the two propositions is true, the other must be false. They must have opposite truth-values. And again, adding a negative particle does not change the situation. 'Few sharks are not predators' has the opposite meaning of 'Many sharks are not predators'. These two propositions, like the others, must have opposite truth-values.

### **Contrary and Sub-Contrary Relations**

There is only one contrary relation on the basic Square, and no sub-contrary relations. However, remember that every statement on the Square is related to every other statement by some relation or other. In fact all of the relations that are not shown on the basic Square are either contraries or sub-contraries. There are too many to consider them all; but let's look at the most important ones.

A and E statements are contraries. If 'All sharks are predators' is true, then 'No sharks are predators' must be false; and vice versa, if 'No sharks are predators' is true, 'All sharks are predators' must be false. Both statements cannot be true. But in fact both statements are false. The great white shark is a predator, but the whale shark is not. Hence 'All sharks are predators' is false, but so is 'No sharks are predators'.

I and O statements are sub-contraries. Both can be true. 'Some sharks are predators' is true, since the great white shark is a predator. But 'Some sharks are not predators' is also true, since the whale shark is not a predator. Yet, both statements cannot be false. If 'Some sparrows are parakeets' is false, then 'Some sparrows are not parakeets' must be true.

In the five-tiered system, the only contrary relation that cannot be derived is the one between the T and D statements, so it is worth a moment to verify that the relation is correct. It is certainly true that both cannot be true. If 'Most Sonnets are Petrarchan sonnets' is true, then 'Most Sonnets are not Petrarchan sonnets' must be false. However, while it is tempting to suppose that T and D statements ought to be contradictories rather than merely contraries, this turns out to be an error. Suppose that all of the sonnets in the world were in fact equally divided between Petrarchan sonnets on the one hand, and other types of sonnets (such as Shakespearean and Spenserian sonnets) on the other hand, so that in fact exactly half of all sonnets are Petrarchan. Then the proposition 'Most Sonnets are Petrarchan sonnets' would be false, since 'most' means more than half; and it is not the case that more than half of all sonnets are Petrarchan. But the proposition 'Most Sonnets are not Petrarchan sonnets' would also be false, for exactly the same reason. Hence, any time we have a class that can be divided exactly in half, it is possible for both T and D statements to be false—though of course it is never possible for them both to be true. Hence they are not contradictories, but only contraries.

Many people may think it is messy to have a contrary relation with no corresponding sub-contrary. The corresponding sub-contraries to the T and D statements are '(At least) 50% of S are P' and '(At least) 50% of S are not P', and these statements could have been added to the Square of Opposition, creating a system with six quantification levels instead of five. But '50%

of' is not a simple quantifier, since it makes use of a numeric value. Systems of proportional quantifiers with numeric proportions have been developed, but the complexities involved are great enough that they should not be inflicted on students learning syllogisms for the first time.

**Using the Square of Opposition to Draw Inferences**

The immediate inferences allowed us to draw conclusions about the truth or falsity of one statement, given the truth or falsity of another statement. The relations on the Square of Opposition can be used in the same way.

*Examples:*

If 'All A are B' is true, then 'Some A are not B' is false, since the two statements are contradictories.
If 'Most C are not D' is false, then 'No C are D' is also false, since 'No C are D' implies 'Most C are not D'.
If 'Many E are F' is true, then 'Most E are F' is undetermined. 'Most E are F' implies 'Many E are F', but the implication relation does not give us the right kind of information to draw an inference to a T statement, given only the truth of a K statement.

*Exercises:*

A. Fill in the blank with 'true', 'false', or 'undetermined'.

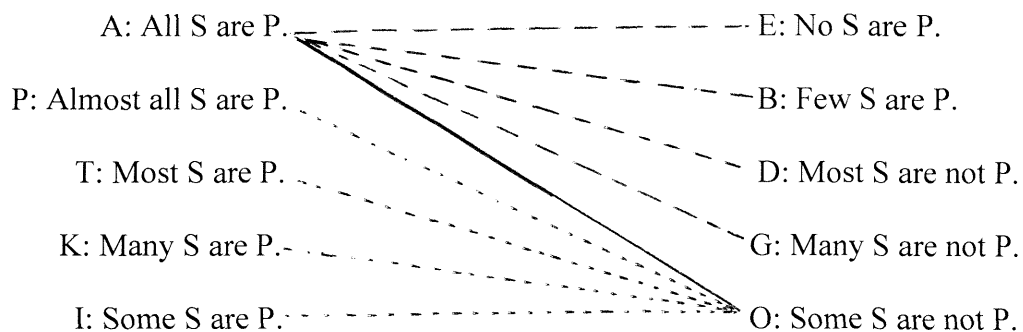
1. 'Many A are not B' is false, so 'Almost all A are B' is \_\_\_\_\_.
2. 'Few C are D' is true, so 'Most C are not D' is \_\_\_\_\_.
3. 'Some E are F' is false, so 'Most E are F' is \_\_\_\_\_.
4. 'Most G are H' is true, so 'Most G are not H' is \_\_\_\_\_.
5. 'All I are J' is false, so 'Some I are not J' is \_\_\_\_\_.
6. 'Most non-K are not L' is true, so 'Few non-K are L' is \_\_\_\_\_.
7. 'Most M are not non-N' is false, so 'Most M are non-N' is \_\_\_\_\_.
8. 'Some non-O are non-P' is true, so 'No non-O are non-P' is \_\_\_\_\_.
9. 'Almost all Q are R' is false, so 'Many Q are R' is \_\_\_\_\_.
10. 'Few non-S are T' is true, so 'Many non-S are T' is \_\_\_\_\_.

B. Fill in the blank with 'true', 'false', or 'undetermined'.

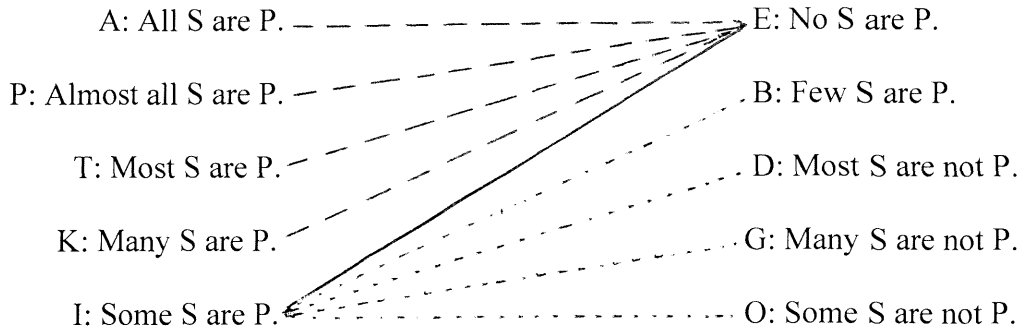
1. 'Most red-colored fruit are not apples' is true, so 'Most red-colored fruit are apples' is \_\_\_\_\_.
2. 'Some Americans are people uninterested in politics' is false, so 'Almost all Americans are people uninterested in politics' is \_\_\_\_\_.
3. 'All doctors are concerned citizens' is true, so 'Some doctors are not concerned citizens' is \_\_\_\_\_.
4. 'Few senior citizens are persons without heart trouble' is false, so 'Many senior citizens are persons without heart trouble' is \_\_\_\_\_.
5. 'All patriotic Americans are communist sympathizers' is true, so 'Many patriotic Americans are communist sympathizers' is \_\_\_\_\_.

### The Full Square of Opposition?

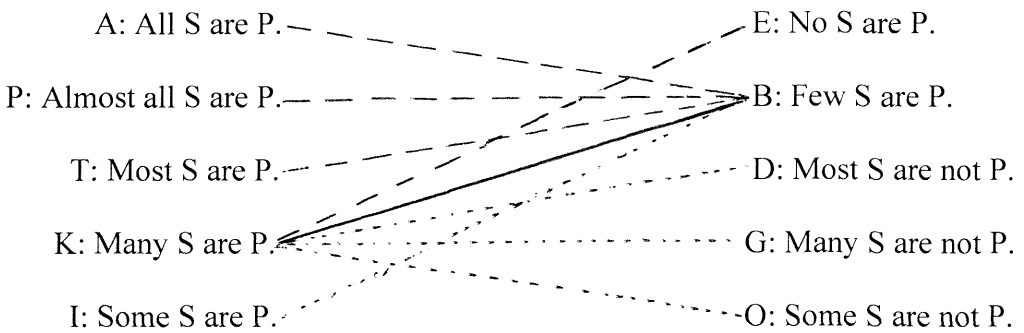
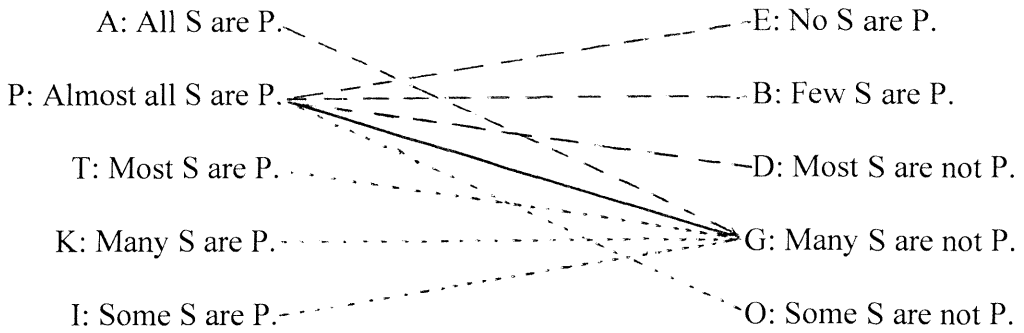
A full Square of Opposition would be a Square on which the relation of every statement to every other statement was explicitly shown. However, such a Square would be difficult to draw, and it would not be particularly useful as a reference tool, even if it *could* be drawn. I find it is more helpful to think of the full Square of Opposition as a group of Japanese fans that can be folded or unfolded as needed. The contradictory relations (represented by solid lines) are the fan when it is closed. Opening the fan upward reveals the contrary relations; opening the fan downward reveals the sub-contrary relations. The A to O relation is one such fan:



The A and O statements are contradictory to each other, but all the relations above that line are contraries; all the relations below that line are sub-contraries. The mirror image of the A to O relation is the I to E relation. These are also contradictories. But again, opening the fan upward reveals the contraries, while opening the fan downward reveals the sub-contraries.



The P to G relation and the K to B relations produce similar fans:



*Exercises:*

A. Fill in the blank with 'true', 'false', or 'undetermined'.

1. 'All A are B' is true, so 'Most A are not B' is \_\_\_\_\_.
2. 'All C are D' is false, so 'Few C are D' is \_\_\_\_\_.

3. 'Almost all E are F' is true, so 'No E are F' is \_\_\_\_\_.
4. 'No G are H' is false, so 'Most G are H' is \_\_\_\_\_.
5. 'No I -are J' is true, so 'Many I are J' is \_\_\_\_\_.
6. 'Many non-K are L' is false, so 'Most non-K are not L' is \_\_\_\_\_.
7. 'Most M are non-N' is true, so 'Many M are not non-N' is \_\_\_\_\_.
8. 'Most non-O are not P' is false, so 'Some non-O are P' is \_\_\_\_\_.
9. 'Many non-Q are R' is true, so 'Many non-Q are not R' is \_\_\_\_\_.
10. 'Some S are non-T' is false, so 'Some S are not non-T' is \_\_\_\_\_.

*B. Fill in the blank with 'true', 'false', or 'undetermined'.*

1. 'All red-colored fruit are apples' is false, so 'No red-colored fruit are apples' is \_\_\_\_\_.
2. 'Most Americans are people interested in politics' is true, so 'Few Americans are people interested in politics' is \_\_\_\_\_.
3. 'Some doctors are not concerned citizens' is false, so 'Few doctors are concerned citizens' is \_\_\_\_\_.
4. 'Most overweight men are persons with heart trouble' is true, so 'Many overweight men are not persons with heart trouble' is \_\_\_\_\_.
5. 'Many patriotic Americans are communist sympathizers' is false, so 'Many patriotic Americans are not communist sympathizers' is \_\_\_\_\_.

**Combining Immediate Inferences with the Square of Opposition**

Remember that two propositions can be compared on the Square of Opposition only if they have exactly the same subject, and exactly the same predicate. 'Most A are B' and 'Most B are not A' cannot be compared to each other, since the subject of the first is not the same as the subject of the second; and likewise for their predicates. Thus, the truth-value of one proposition may still leave the truth-value of another proposition undetermined. In this case there is no way to move the terms around so that they do match. But in other cases the immediate inferences, conversion, contraposition, and obversion, may allow us to manipulate the terms in a way that produces a match. Hence, some pairs of statements may be related to each other in complex and

subtle ways. Discovering how they are related may require both the immediate inferences and the relations on the Square of Opposition.

*Examples:*

If 'Few A are non-B' is true, then 'Many A are B' is true.

'Few A are non-B' is true, so

by obversion

'Almost all A are B' is true, so

by implication

'Many A are B' is true.

If 'Almost all non-C are non-D' is false, then 'Some D are not C' is true.

'Almost all non-C are non-D' is false, so

by contradiction

'Many non-C are not non-D' is true, so

by implication

'Some non-C are not non-D' is true, so

by contraposition

'Some D are not C' is true.

In the most complex cases, it may take up to four steps to draw an inference from one statement to another. However, no derivation takes more than four steps, and the ones that do are easy to spot. Both sentences will have non-intermediate quantifiers, i.e. 'all', 'no', or 'some'. In addition, they will have matching predicates, but non-matching subjects.

*Example:*

If 'Some non-A are non-B' is false, then 'Some A are non-B' is true.

'Some non-A are non-B' is false, so

by conversion

'Some non-B are non-A' is false, so

by obversion

'Some non-B are not A' is false, so

by sub-contrariety

'Some non-B are A' is true, so

by conversion

'Some A are non-B' is true.

*Exercises:*

*A. Fill in the blank with 'true', 'false', or 'undetermined'.*

1. 'Most non-A are B' is true, so 'Most non-A are non-B' is \_\_\_\_\_.
2. 'Almost all C are non-D' is false, so 'Some C are not D' is \_\_\_\_\_.
3. 'Few E are non-F' is true, so 'Some F are E' is \_\_\_\_\_.
4. 'Many G are H' is false, so 'All non-H are non-G' is \_\_\_\_\_.
5. 'All I are J' is true, so 'All non-I are J' is \_\_\_\_\_.
6. 'Many K are not L' is false, so 'Many K are not non-L' is \_\_\_\_\_.
7. 'Most non-M are not non-N' is true, so 'Most non-M are not N' is \_\_\_\_\_.
8. 'No O are P' is false, so 'Most P are O' is \_\_\_\_\_.

9. 'Some Q are not non-R' is true, so 'Most R are non-Q' is \_\_\_\_\_.

10. 'Some non-S are non-T' is false, so 'All T are S' is \_\_\_\_\_.

B. Fill in the blank with 'true', 'false', or 'undetermined'.

1. 'Some swans are flightless birds' is false, so 'Some flightless birds are not swans' is \_\_\_\_\_.

2. 'Many celebrities are not polite people' is true, so 'No impolite people are celebrities' is \_\_\_\_\_.

3. 'Most politicians are dishonest people' is false, so 'Many honest people are politicians' is \_\_\_\_\_.

4. 'No persons without a secure job are persons safe from a financial recession' is true, so 'Many persons who are safe from a financial recession are persons with a secure job' is \_\_\_\_\_.

5. 'All football players are overpaid workers' is false, so 'Most underpaid workers are football players' is \_\_\_\_\_.

### Notes

<sup>1</sup>Other relations are logically possible, though no other relations are important in this system. For a discussion of the logically possible relations, see Mark Brown, "Generalized quantifiers and the square of opposition," *The Notre Dame Journal of Formal Logic*, Vol. 25, No. 4 (Oct, 1984), pp. 303-322.

<sup>2</sup>The reason for this incapacity has to do with the mathematics of symbolic logic, and specifically with the truth-functional definition of the implication operator. However this is a topic that cannot be conveniently treated here. For a discussion of this issue, please see R. B. Angell, "Truth-functional conditionals and modern vs. traditional syllogistic logic," *Mind*, Vol. 95 (1986), pp. 210-223.

<sup>3</sup>Obviously this relatively brief discussion merely introduces, and does not by any means exhaust this complex and important topic. Any author who discusses syllogistic logic must discuss the consequences of existential import. R. B. Angell, "The Boolean interpretation is wrong," *Readings on Logic*, ed. I. M. Copi and J. A. Gould, MacMillan Publishing Co., New York, 1972, defends the view that I hold. J. C. Shepherdson, "On the interpretation of Aristotelian syllogistic," *Journal of Symbolic Logic*, Vol. 21 (1956), pp. 137-147, defends the Boolean view. T. G. Nedzyski, "Quantification, domains of discourse, and existence," *The Notre Dame Journal of Formal Logic*, Vol. 20, No 1 (Jan, 1979), pp. 130-140, defends the view that I have rejected, that the Aristotelian view can be rescued by an appeal to "domains of discourse." See also, Karl Lambert, "Existential import revisited," *The Notre Dame Journal of Formal Logic*, Vol. 4, No. 4 (Oct, 1963), pp. 288-292.