Section 6.6: Trigonometric Equations

Thursday, March 06, 2014
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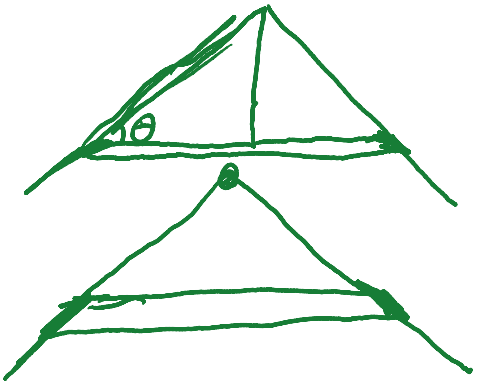
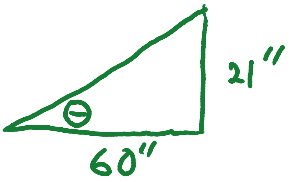
Goal: To solve these things!





Find θ.

$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{21}{60}\right)$

$\theta = \tan^{-1}\left(\frac{21}{60}\right)$

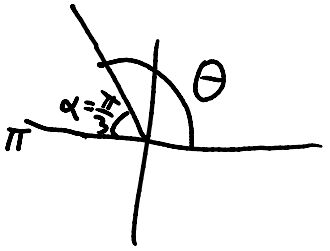
$\theta = 19.3^\circ$

ex Solve (x is in radians)

$$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$



$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Q II

$$x = \frac{\pi}{3} \quad \text{or} \quad x = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$$

$x = \frac{\pi}{3}$ or $x = \frac{2\pi}{3}$, $0 \le x < 2\pi$

all solutions

$x = \frac{\pi}{3} + 2k\pi$ or $x = \frac{2\pi}{3} + 2k\pi$

where k = integer $= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

ex Solve exactly for $0 \le x < 2\pi$

a) factoring

$$2\sin x \cos x = \sqrt{3}\sin x$$

$-\sqrt{3}\sin x \quad -\sqrt{3}\sin x$

$2\sin x\cos x - \sqrt{3}\sin x = 0$

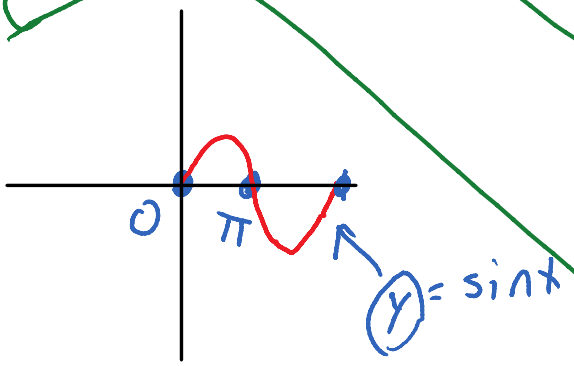
$\sin x\left(2\cos x - \sqrt{3}\right) = 0$

$\sin x = 0$ or $2\cos x - \sqrt{3} = 0$

$x = 0, \pi$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$



$\frac{12\pi}{6}$

$x = \frac{\pi}{6}, \quad x = 2\pi - \frac{\pi}{6}$

$x = \frac{11\pi}{6}$

$x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$

b) taking roots

$2\sin^2 x - 1 = 0$

$+1 \quad +1$

$\frac{2\sin^2 x}{2} = \frac{1}{2}$

$$\sqrt{\sin^2 x} = \pm\sqrt{\frac{1}{2}}$$

$$\sin x = \pm \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$$\sin x = -\frac{\sqrt{2}}{2} \quad \text{or} \quad \sin x = \frac{\sqrt{2}}{2}$$

$$\alpha = \frac{\pi}{4} \qquad x = \frac{\pi}{4}, \quad \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

(ex) solve for $0 \le x < 360^\circ$

a) $2\cot^2 x - 7\cot x + 3 = 0$

$-1\cot x$

$(2\cot x - 1)(\cot x - 3) = 0$

$-6\cot x$

$2\cot x - 1 = 0$ or $\cot x - 3 = 0$

$\cot x = \frac{1}{2}$ or $\cot x = 3$

$\tan x = 2$ or $\tan x = \frac{1}{3}$

$x = \tan^{-1}(2)$ $\quad$ $x = \tan^{-1}\left(\frac{1}{3}\right)$

$\approx 63.4^\circ$ $\quad$ $\approx 18.4^\circ$

or $x \approx 180^\circ + 63.4^\circ = 243.4^\circ$ $\quad$ or $x \approx 198.4^\circ$

b) $2\cos^2 x - 5\cos x - 5 = 0$

$au^2 + bu + c = 0$

$u = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 2$ $b = -5$ $c = -5$

$$\cos x = \frac{5 \pm \sqrt{25 - 4(2)(-5)}}{4}$$

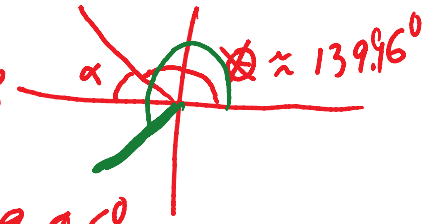
$$\cos x = \frac{5 \pm \sqrt{65}}{4}$$

$$\cos x = \frac{5 - \sqrt{65}}{4} \quad \text{or} \quad \cos x = \frac{5 + \sqrt{65}}{4} > 1$$

no solution

$$x = \cos^{-1}\left(\frac{5 - \sqrt{65}}{4}\right)$$

$$\approx 139.96^\circ$$



$\alpha = 180^\circ - 139.96^\circ$

$= 40.04$

$x \approx 180 + 40.04$

$= 220.04^\circ$

c) $\cos(4x) = -\frac{\sqrt{2}}{2}$

$\alpha = 45^\circ$

note: $y = \cos 4x$ has period $\frac{2\pi}{4} = \frac{\pi}{2} = 90^\circ$

$\alpha = 45^\circ$

$4x = 135^\circ$ or $4x = 225^\circ$

$x = \frac{135^\circ}{4}$ or $x = \frac{225^\circ}{4}$

$x = 33.75^\circ$ or $x = 56.25^\circ$

or $+90^\circ$ each time

$= 123.75^\circ$ or $= 146.25^\circ$

$= 213.75^\circ$ or $= 236.25^\circ$

$= 303.75^\circ$ or $= 326.25^\circ$

d) $\sin x + 2\cos x = 1$ (see video labeled # 51)

$(2\cos x)^2 = (1 - \sin x)^2$

$$4\cos^2 x = 1 - 2\sin x + \sin^2 x$$

$$4(1 - \sin^2 x) = 1 - 2\sin x + \sin^2 x$$

$$4 - 4\sin^2 x = 1 - 2\sin x + \sin^2 x$$

$$0 = -3 - 2\sin x + 5\sin^2 x$$

$$0 = 5\sin^2 x - 2\sin x - 3$$

$$0 = (5\sin x + 3)(\sin x - 1)$$

$$\sin x = \frac{-3}{5} \quad \text{or} \quad \sin x = 1$$

$$x = \frac{\pi}{2}$$

$$x = \sin^{-1}\left(\frac{-3}{5}\right)$$

$$\approx -0.644$$

$$\alpha = 0.644$$

$$x \approx \pi + .644$$

$$\approx 5.786$$ ← doesn't check

or

$$x \approx 2\pi - .644$$

$$\approx 5.639$$

ex Solve for $0 \le x < 2\pi$

(ex) Solve for $0 \le x < 2\pi$

a) $\cos 2x = 2\cos x - 1$

$2\cos^2 x - 1 = 2\cos x - 1$

$+1 \quad -2\cos x \quad +1$

$-2\cos x$

$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

$2\cos x = 0 \quad \text{or} \quad \cos x - 1 = 0$

$\cos x = 0 \quad \text{or} \quad \cos x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = 0$

b) $\sec^2 x + \sqrt{3}\sec x - 2\sec x - 2\sqrt{3} = 0$

$\sec x(\sec x + \sqrt{3}) - 2(\sec x + \sqrt{3}) = 0$

$(\sec x - 2)(\sec x + \sqrt{3}) = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\sec x = -\sqrt{3}$

$\cos x = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\cos^{-1}(\cos x) = \cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$x = \cos^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$x \approx 2.19$

$\alpha = \pi - 2.19$

$x \approx 4.09$