Section 7.4 Part II: Trigonometric Form of Complex Numbers

Thursday, April 10, 2014
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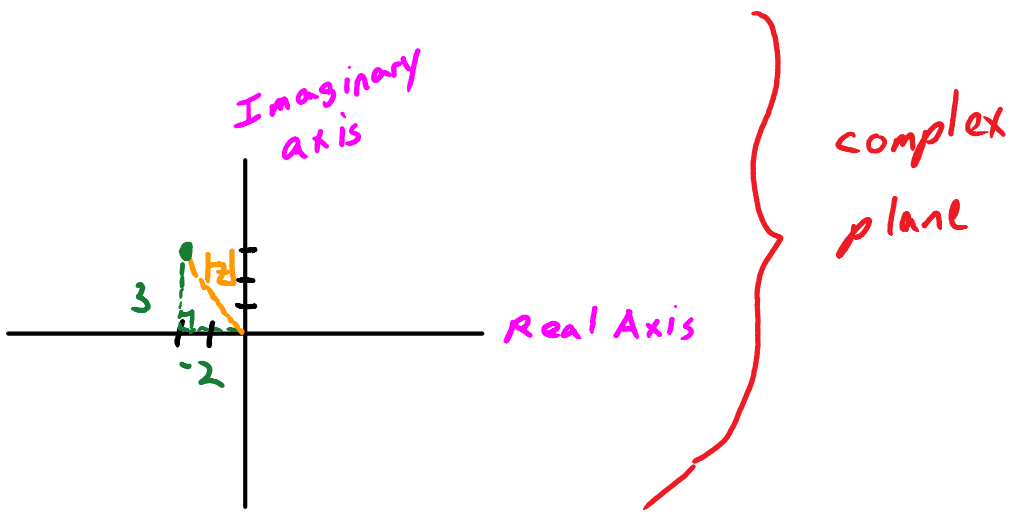
Goal:

1. To plot a complex number in the complex plane.
2. To convert between standard form and trig form.
3. To multiply and divide complex numbers in trig form.

ex) consider $z = {}^-2 + 3i$. Plot z in complex plane

$z = {}^-2 + 3i = (-2, 3)$



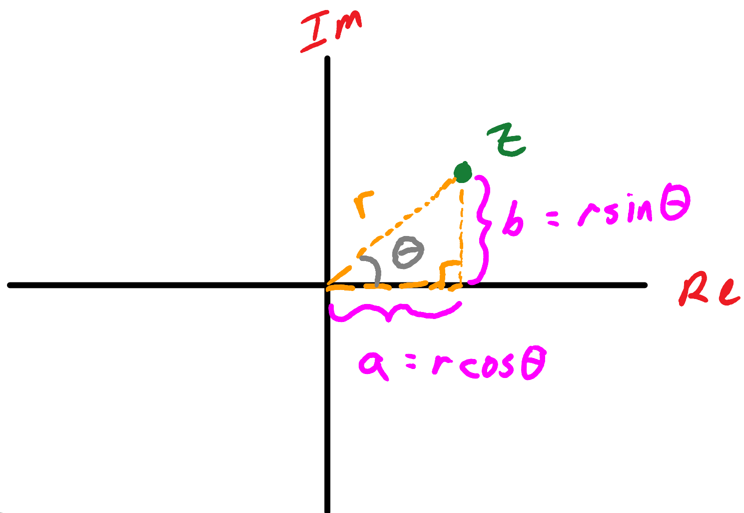
$$|\underbrace{-2+3i}_{z}| = \sqrt{(-2)^2 + (3)^2}$$

$$= \sqrt{13}$$

Def of $|z|$

If $z = a + bi$. Then $|z| = \sqrt{a^2 + b^2}$

Trig form of $z = a + bi$



$$\boxed{z = r\cos\theta + i\, r\sin\theta}$$ ← trig form

$$z = r\underbrace{(\cos\theta + i\sin\theta)}_{r\,\text{cis}\,\theta}$$

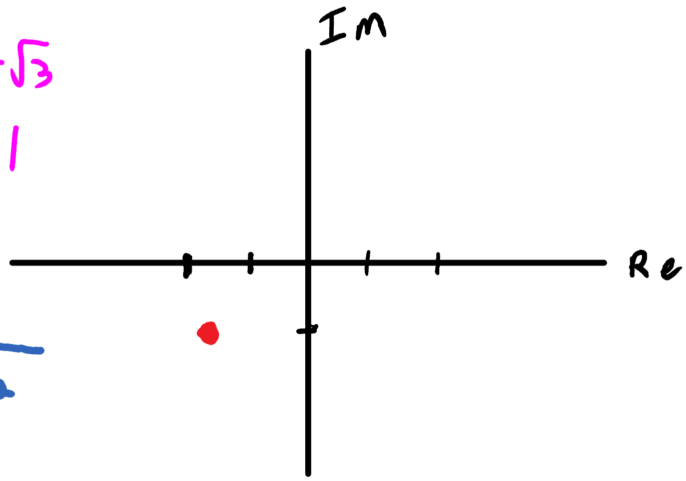
Note: $\tan\theta = \dfrac{b}{a}$

ex) convert to trig form

a) $z = -\sqrt{3} - 1i$

$a = -\sqrt{3}$

$b = -1$



$r = \sqrt{(-\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$= 2$

$\tan\theta = \dfrac{-1}{-\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$\tan\theta = \dfrac{\sqrt{3}}{3}$

$\alpha = 30^\circ$

$\theta = 180^\circ + 30^\circ = 210^\circ$

$z = r\cos\theta + i r \sin\theta$

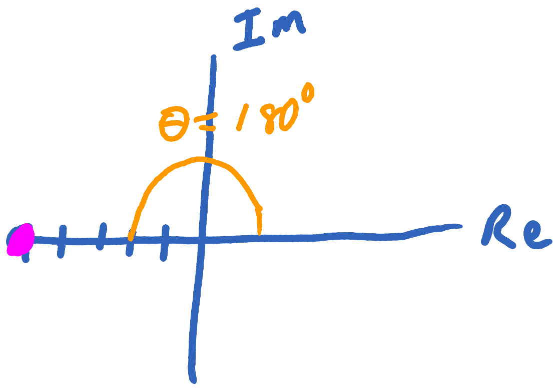
$= 2\cos 210^\circ + 2i\sin 210^\circ$

$$= 2 \text{ cis } 210^\circ$$

b) $z = -5$

$z = -5 + 0i$

$z = (-5, 0)$



$|z| = r = |-5| = 5$

$\theta = 180^\circ$

$$z = 5 \text{ cis } 180^\circ$$

Product of Two complex #'s

$$z_1 = r_1(\cos\theta_1 + i\sin\theta_1) = r_1 \text{ cis } \theta_1$$

$$z_2 = r_2(\cos\theta_2 + i\sin\theta_2) = r_2 \text{ cis } \theta_2$$

$$z_1 z_2 = r_1 r_2(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2\left[\cos\theta_1\cos\theta_2 + \underbrace{i\sin\theta_2\cos\theta_1 + i\sin\theta_1\cos\theta_2}_{} + \overset{(-1)}{i^2}\sin\theta_1\sin\theta_2\right]$$

$$= r_1 r_2 \left[\underbrace{\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2}_{\cos(\theta_1+\theta_2)} + i\left(\underbrace{\sin\theta_1 \cos\theta_2 + \sin\theta_2 \cos\theta_1}_{\sin(\theta_1+\theta_2)}\right)\right]$$

$$= r_1 r_2 \left(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\right)$$

$$\boxed{z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)}$$

ex Multiply

a) $(2 \cdot \operatorname{cis} 30^\circ)(3 \cdot \operatorname{cis} 225^\circ)$

$= 6 \operatorname{cis}(30^\circ + 225^\circ)$

$= 6 \operatorname{cis}(255^\circ)$

b) $5\left[\cos\left(\frac{2\pi}{5}\right) + i \sin\left(\frac{2\pi}{5}\right)\right] \cdot 2\left[\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right]$

$$= 10\left[\cos\left(\frac{2\pi}{5}+\frac{2\pi}{5}\right)+i\sin\left(\frac{2\pi}{5}+\frac{2\pi}{5}\right)\right]$$

$$= 10\left[\cos\frac{4\pi}{5}+i\sin\frac{4\pi}{5}\right]$$

$$= 10\operatorname{cis}\left(\frac{4\pi}{5}\right)$$

Division

$$\frac{z_1}{z_2} = \frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2}$$

$$= \boxed{\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)}$$

ex

$$\frac{32 \operatorname{cis} 30^\circ}{4 \operatorname{cis} 150^\circ}$$

$$= 8 \operatorname{cis}(30^\circ - 150^\circ)$$

$= 8 \text{ cis}(-120°)$

$= \underbrace{8 \text{ cis}(240°)}$

write in standard form

$= 8(\cos 240° + i \sin 240°)$

$= 8\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$= -4 - 4i\sqrt{3}$