Section 8.5: Polar Coordinates

Friday, March 28, 2014
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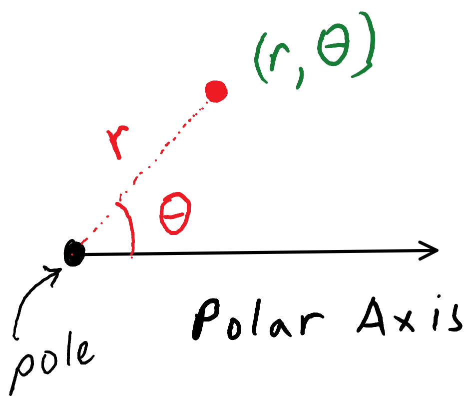
Goals:

1. To convert between polar and rectangular coordinates.
2. To graph in polar coordinates.

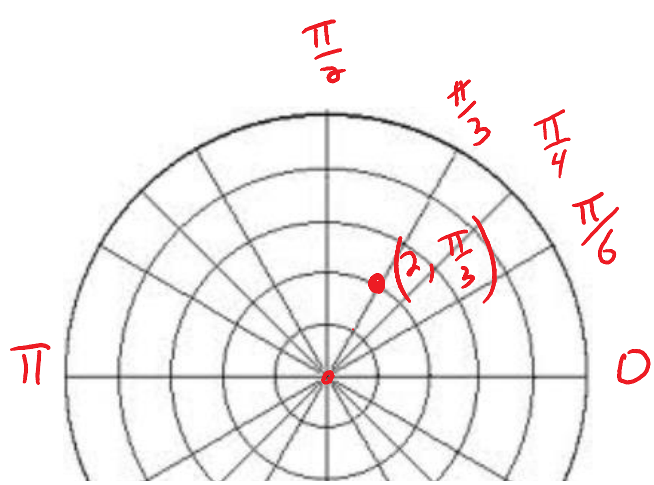
Another system for locating points using a radius and an angle (r, θ)



ex plot the points

a) $\left(\overset{r}{2}, \overset{\theta}{\frac{\pi}{3}}\right)$

$$\left(2, \frac{\pi}{3}\right) = \left(2, \frac{\pi}{3} + \overset{\frac{6\pi}{3}}{2\pi}\right)$$



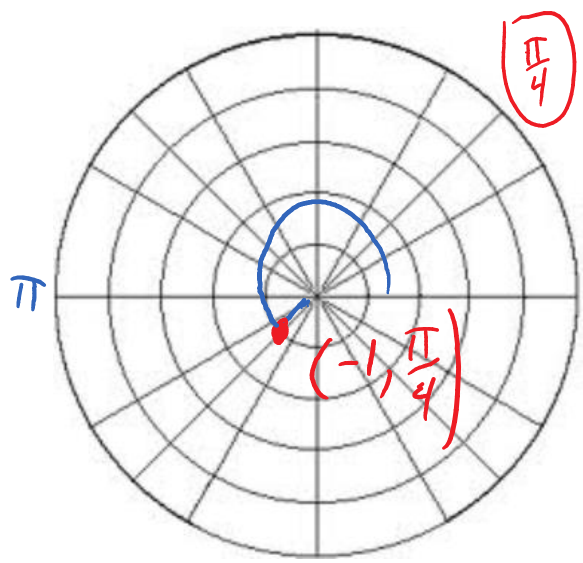
$$\left(2, \frac{\pi}{3}\right) = \left(2, \frac{\pi}{3} + 2\pi\right)$$

$$= \left(2, \frac{7\pi}{3}\right)$$

b) $\left(\overset{r}{-1}, \overset{\theta}{\frac{\pi}{4}}\right)$

$$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$\left(1, \frac{5\pi}{4}\right)$$



Coordinate conversions

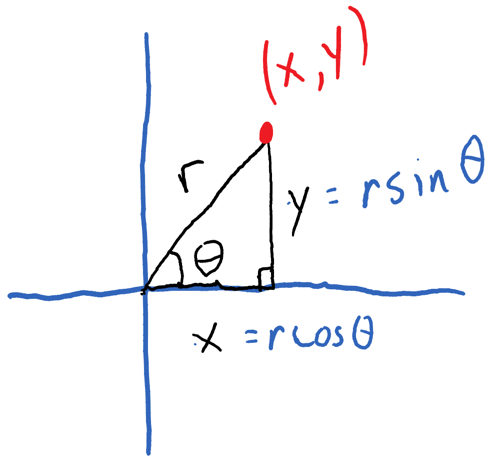
not new

$$r^2 = x^2 + y^2$$ pyth. Th.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$



ex convert to rectangular

$$\text{a)}\quad (r,\theta) = \begin{array}{c} r \quad \theta \\ \left(5, \frac{\pi}{6}\right) \end{array}$$

$$x = r\cos\theta = 5\cos\frac{\pi}{6} = \frac{5\sqrt{3}}{2}$$

$$y = r\sin\theta = 5\sin\frac{\pi}{6} = \frac{5}{2}$$

$$\left(\frac{5\sqrt{2}}{2}, \frac{5}{2}\right)$$

b) $r = \cos\theta$

$$r \cdot r = r\cos\theta$$

$$r^2 = r\cos\theta$$

$$x^2 + y^2 = x$$

$$\left(x^2 - 1x + \frac{1}{4}\right) + y^2 = 0 + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4} \leftarrow \left(\frac{1}{2}\right)^2$$

std form of a circle!

$$(x-h)^2 + (y-k)^2 = r^2$$

$C\left(\frac{1}{2}, 0\right)$

radius $= \frac{1}{2}$

c) $r = \cos\theta + 2\sin\theta$

$$r \cdot r = r(\cos\theta + 2\sin\theta)$$

$$r^2 = \underbrace{r\cos\theta}_{} + 2\underbrace{r\sin\theta}_{}$$

$$x^2+y^2 = \cancel{x} + 2y$$

Exercise: show this a circle by writing in std. form

(ex) convert to polar

a) $(x,y) = \underbrace{(\overset{x}{-1}, \overset{y}{1})}_{}$

we want $\underline{(r, \theta)}$

need r and θ

$$r^2 = x^2+y^2$$

$$r^2 = (-1)^2+(1)^2$$

$$r^2 = 2$$

$$r = \sqrt{2}$$

$$\tan\theta = \frac{1}{-1} = -1$$

$$-45^\circ$$

$$\alpha = 45^\circ$$

$$\theta = 180^\circ - 45^\circ = 135^\circ$$

$$\theta = 180^\circ - 45^\circ = 135^\circ$$

$$(\sqrt{2}, 135^\circ)$$

b) $x = -4$

$$r\cos\theta = -4$$

$$r = \frac{-4}{\cos\theta}$$

$$r = -4\sec\theta$$

c) $y^2 = 4y$

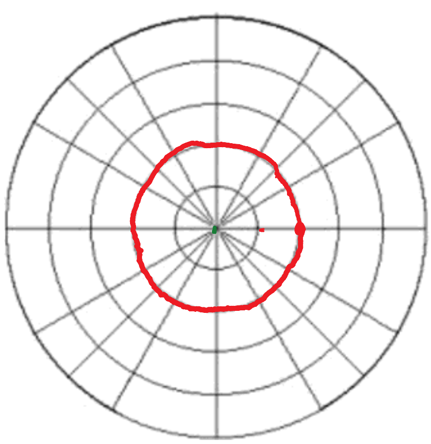
$$(r \cdot \sin\theta)^2 = 4(r\sin\theta)$$

$$r^2\sin^2\theta = 4r\sin\theta$$

$$r^2 \sin^2\theta = 4r\sin\theta$$

ex) Graph

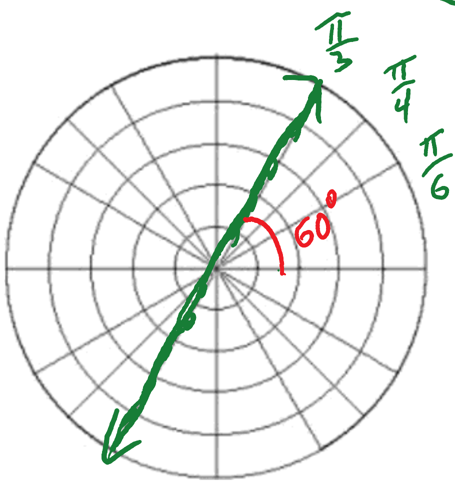
a) $r = 2$



$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 = 4$$

b) $\theta = \frac{\pi}{3}$



convert to rectangular

$\tan(\theta) = \tan\left(\frac{\pi}{3}\right)$

$\frac{y}{x} = \sqrt{3}$

$\boxed{y = \sqrt{3}\,x}$

Special Polar Graphs

Limacon

symm to x-axis

sym. to y-axis

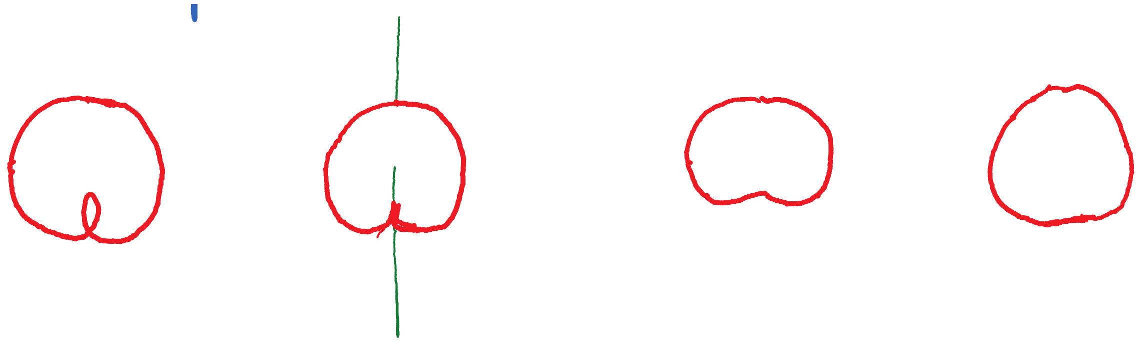
$\boxed{r = a \pm b\cos\theta}$ or $\boxed{r = a \pm b\sin\theta}$, a and b constants

$0 < \left|\frac{a}{b}\right| < 1$ inner loop

$\left|\frac{a}{b}\right| = 1$ cardioid

$1 < \left|\frac{a}{b}\right| < 2$ dimple

$\left|\frac{a}{b}\right| > 2$



$r = a + b\sin\theta$

Rose Curves

$\boxed{r = a\sin(n\theta)}$ or $\boxed{r = a\cos(n\theta)}$, where

n is a counting number > 1. $(n = 2, 3, 4, \dots)$

If n is odd $\rightarrow$ n petals

n is even $\rightarrow$ $2n$ petals

(ex) Graph

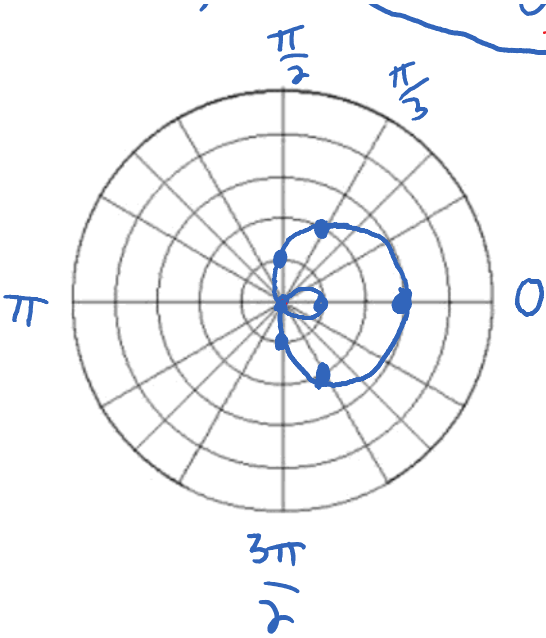
symm. x-axis

a) $r = 1 + 2\cos\theta$

0, a, b, -1

$\frac{\pi}{2}$ $\frac{\pi}{2}$

$0 < \left|\frac{1}{2}\right| = \frac{1}{2} < 1$



$$0 < \left|\frac{1}{2}\right| = \frac{1}{2} < 1$$

r	θ
3	0
2	$\pi/3$
1	$\pi/2$
0	$\frac{2\pi}{3}$
-1	π
1	$3\pi/2$
3	2π

(-1)

$r = 1 + 2\cos\theta$

$1 + 2\left(\frac{1}{2}\right)$

$$0 = 1 + 2\cos\theta$$

$$\cos\theta = -\frac{1}{2}$$

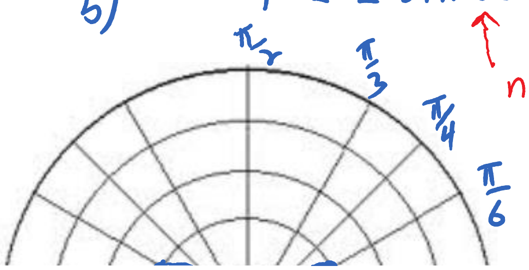
$$\alpha = \frac{\pi}{3}$$

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

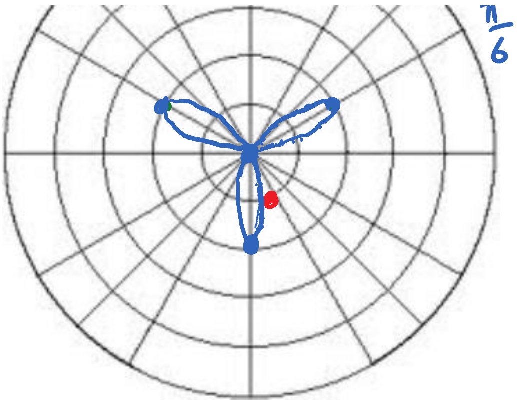
sym. to y-axis

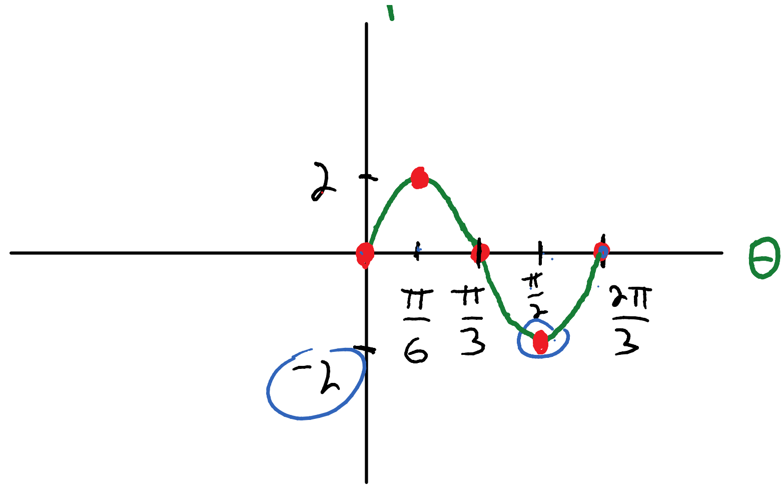
b) $r = 2\sin 3\theta$

$n = 3 \rightarrow 3$ odd $\rightarrow 3$ petals



r

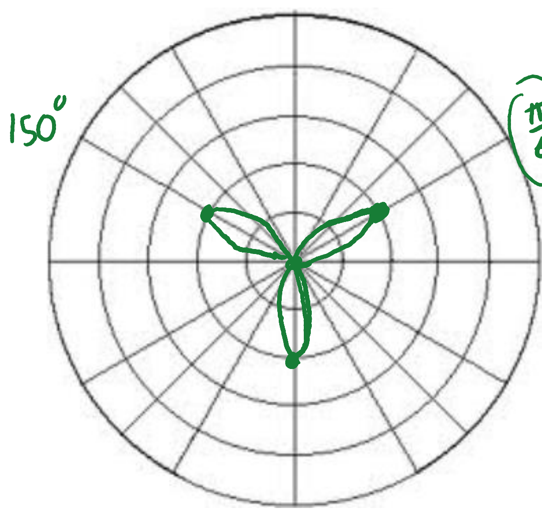




$$r = 2\sin 3\theta$$

$$\frac{2\pi}{6\cancel{3}} + \frac{\pi}{6}$$

$$p = \frac{2\pi}{3}$$



$\frac{\pi}{6} = 30°$

$$r = 2\sin \underbrace{3\theta}_{\frac{\pi}{2}}$$

which θ will produce $r = 2$ (biggest r can be)

$$3\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{6}$$

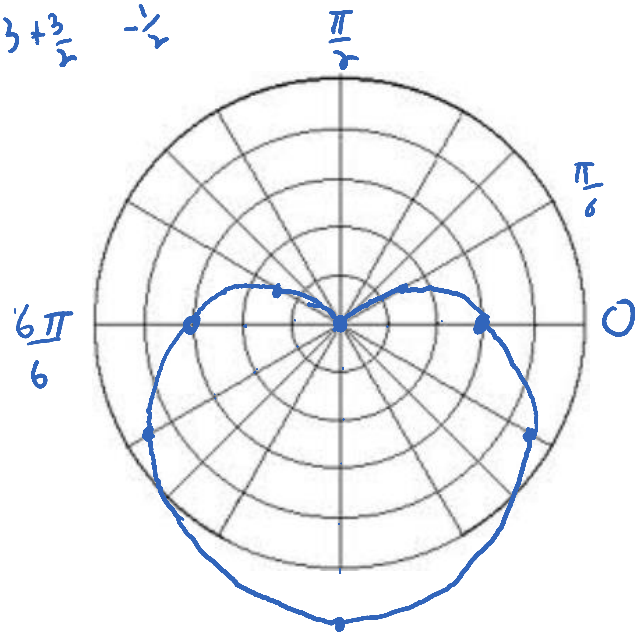
$$\frac{360°}{3} = 120°$$

ex Graph

a) $r = 3 - 3\underbrace{\sin\theta}_{-\frac{1}{2}}$

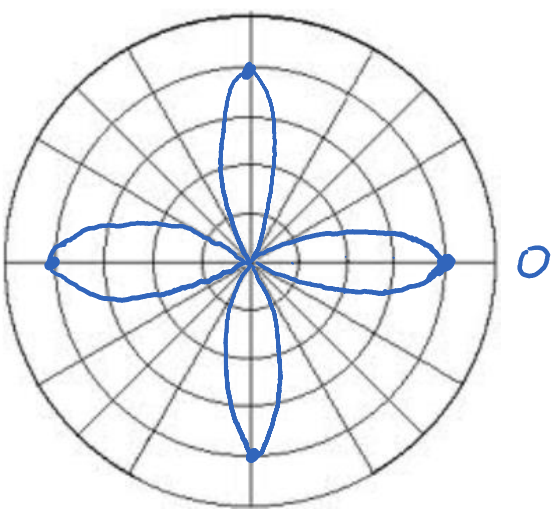
$3 + \frac{3}{2}$

r	θ
3	0
$1.5 = \frac{3}{2}$	$\pi/6$
0	$\frac{\pi}{2}$
4.5	$\frac{7\pi}{6}$
3	π
6	$3\pi/2$
3	2π



b) $r = 4\cos 2\theta$

$n = 2 \rightarrow 4$ petals



$\frac{360^\circ}{4} = 90^\circ$