

# Graphing Techniques

Goal: To graph trig functions of the form  $y = Af(Bx-h)+k$

$$f(x) = \sin x, \text{ or } \cos x, \tan x, \dots$$

Recall: Let A, B, k, h>0 and  $y = f(x)$

Function of the form...	Transforms the graph of $y = f(x)$ ...
$y = f(x)+k$	up k units
$y = f(x)-k$	down k units
$y = f(x-h)$	right h units
$y = f(x+h)$	left h units
$y = Af(x)$	by a vertical stretch/shrink factor of A
$y = f(Bx)$	by a horizontal stretch/shrink factor of $1/B$
$y = -f(x)$	by a reflection across the x-axis

vertical-shift  
horizontal-shift or phase shift

Ex. Graph 1 full period of ...

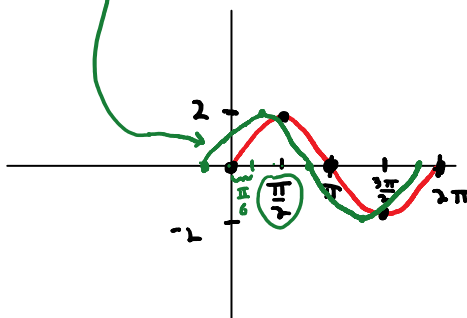
a)  $y = 2 \sin(x + \frac{\pi}{6})$  ← left

helper:  $y = 2 \sin x$

Amp =  $|2| = 2$

P =  $2\pi$

P.S. = left  $\frac{\pi}{6} = -\frac{\pi}{6}$



b)  $y = \cos(2x - \frac{\pi}{2})$

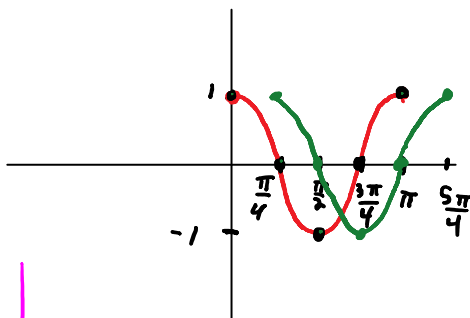
helper:  $y = \cos 2x$

P =  $\frac{2\pi}{2} = \pi$

Amp = 1

$$y = \cos(2x - \frac{\pi}{2})$$

$$= \cos[2(x - \frac{\pi}{4} \cdot \frac{1}{2})]$$



$2x - \frac{\pi}{2} = 0$

$$= \cos\left[2\left(x - \frac{\pi}{2} \cdot \frac{1}{2}\right)\right]$$

$$= \cos\left[2\left(x - \frac{\pi}{4}\right)\right]$$

P.S. =  $\frac{\pi}{4}$

$$2x - \frac{\pi}{2} = 0$$

$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}$$

P.S. (right)

c)  $y = 3 \sin\left(\frac{\pi x}{2} + 1\right) - 2$

helper:  $y = 3 \sin\left(\frac{\pi}{2}x\right)$

$$P = \frac{2\pi}{\pi/2} = 2\pi \cdot \frac{2}{\pi} = 4$$

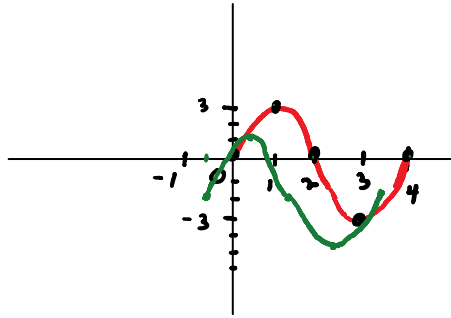
$$\text{Amp} = |3| = 3$$

V.S:  $-2$  (down 2)

P.S:  $\frac{\pi}{2}x + 1 = 0$

$$\frac{\pi}{2}x = -1$$

$$x = -\frac{2}{\pi} \approx -0.636$$



(ex) Let  $y = -3 \cos\left(2x + \frac{\pi}{4}\right) - 5$

helper ①  $y = 3 \cos(2x)$

helper ②  $y = -3 \cos(2x)$

Find

a) Amplitude

$$\text{Amp} = |-3| = 3$$

b) Period

c) phase shift

$$2x + \frac{\pi}{4} = 0 \rightarrow 2x = -\frac{\pi}{4} \rightarrow x = -\frac{\pi}{8}$$

left  $\frac{\pi}{8}$

d) vertical shift

b) Period

$$\frac{2\pi}{2} = \pi$$

d) vertical shift

-5 (down 5)

e) reflect?

reflection across x-axis (if graphing, do this first)

Ex) Graph two periods of  $y = \frac{3}{2} \cot\left(2x - \frac{\pi}{2}\right)$

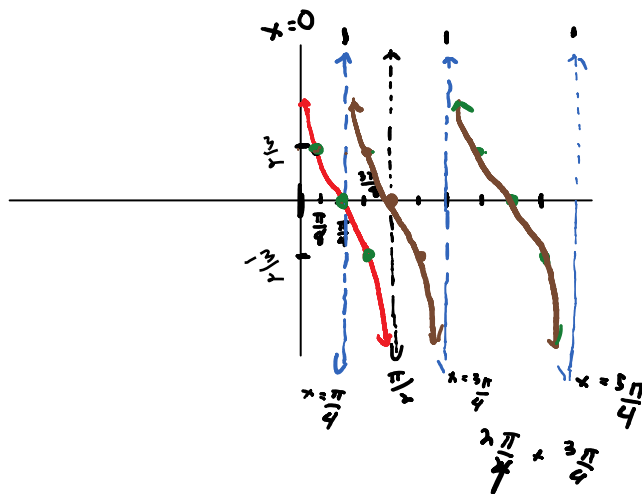
helper:  $y = \frac{3}{2} \cot(2x)$

$$p = \frac{\pi}{2}$$

$$2x - \frac{\pi}{2} = 0$$

$$2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4} \text{ right } \frac{\pi}{4}$$

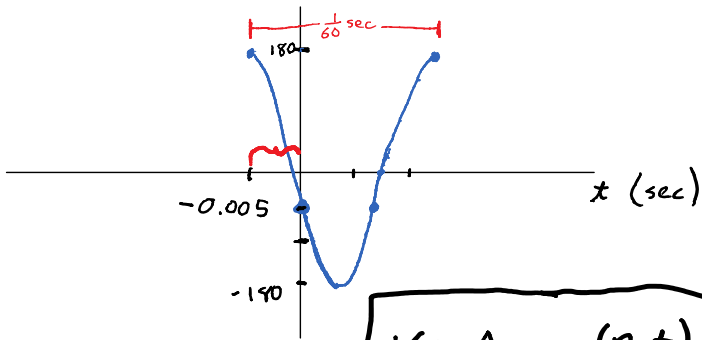


Ex. The graph of voltage from an alternating household circuit is shown below. Find an equation that gives voltage at time t. [Aufmann, number 68 p. 499, 8th ed].

helper

v (volts)





$$A = 180$$

$$B = 120\pi$$

$$\frac{2\pi}{B} = \frac{1}{60}$$

$$V = A \cos(Bt)$$

P.S. -0.005

$$V = 180 \cos[120\pi(t + 0.005)]$$

$$V = 180 \cos\left[120\pi t + \frac{3}{5}\pi\right]$$

Ex. Find the equation of the graph in red

helper

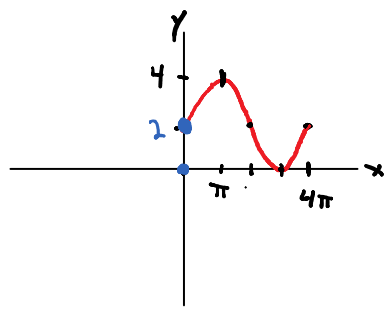
$$y = A \sin(Bx)$$

$$P = \frac{4\pi}{1} = \frac{2\pi}{B}$$

$$B = \frac{2\pi}{4\pi} = \left(\frac{1}{2}\right)$$

$$\text{Amp} = \frac{\text{max} - \text{min}}{2} = 2$$

V.S. +2



$$y = 2 \sin\left(\frac{1}{2}x\right) + 2$$