Section 6.1: Trigonometric Identities

Thursday, February 13, 2014 12:59 PM

Goal: To verify trig identities

Note: For sections 6.1-6.3, the homework is from the etext, which can is accessed from within Webassign.

HW 6.1: 1-69 e.o.o. ("e.o.o." mean every other odd (1, 5, 9,...,69)

A trig identity is an equation that is always true for all relevant x-values

$$\boxed{\sin x = \tan x \cos x} \quad \text{identity}$$

ex $\tan x \cos x = \sin x$ (as long as x isn't an odd multiple of $\frac{\pi}{2}$)

$$\tan x \cos x = \left(\frac{\sin x}{\cancel{\cos x}}\right) \cdot \cancel{\cos x}$$

$$= \sin x$$

Done

Known Identities

$\sin x = \frac{1}{\ }$ $\cos x = \frac{1}{\ }$ $\tan x = \frac{1}{\ }$

Known Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

$$\cos^2 x + \sin^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad \cot^2 x + 1 = \csc^2 x$$

$$\cos^2 x = 1 - \sin^2 x \qquad \tan^2 x = \sec^2 x - 1$$

$$1 - \cos^2 x = \sin^2 x \qquad \tan(x)^2$$

(ex) Verify

a) $\sin x \cot x \sec x = 1$

$$\sin x \cot x \sec x = \cancel{\sin x} \cdot \left(\frac{\cancel{\cos x}}{\cancel{\sin x}}\right) \cdot \left(\frac{1}{\cancel{\cos x}}\right)$$

$$= 1$$

Done ✓ ☺ Q.E.D

b) $$\frac{1}{\sin x} + \frac{3}{\cos x} = \frac{\cos x + 3\sin x}{\sin x \cos x}$$

LCD $= \sin x \cos x$

$$\frac{1}{\sin x} \cdot \frac{\cos x}{\cos x} + \frac{3}{\cos x} \cdot \frac{\sin x}{\sin x} = \frac{\cos x}{\sin x \cos x} + \frac{3\sin x}{\sin x \cos x}$$

$$= \frac{\cos x + 3\sin x}{\sin x \cos x}$$

Done

c) $\dfrac{\sin^2 x - 2\sin x + 1}{\sin x - 1} = (\sin x - 1)\dfrac{(\sin x - 1)}{(\sin x - 1)}$

$$\frac{\sin^2 x - 2\sin x + 1}{\sin x - 1} = \frac{(\sin x - 1)^{\cancel{2}}}{\cancel{(\sin x - 1)}}$$

$$= \sin x - 1$$

$(A - B)^2$

$= A^2 - 2AB + B^2$

$A = \sin x,\ B = 1$

$(\quad)(\quad)$

d) $\sec^2 x + 2\tan x = (\tan x + 1)^2$

$$(\tan x + 1)^2 = \boxed{\tan^2 x + 2\tan x + 1}$$

$$= \boxed{(\tan^2 x + 1) + 2\tan x}$$

$(A + B)^2$

$= A^2 + \boxed{2AB} + B^2$

$= \tan^2 x + 1$

$$= \boxed{(\tan^2 x + 1) + 2\tan x}$$

$$= \sec^2 x + 2\tan x$$

Done

$\sec^2 x = \tan^2 x + 1$

e) $\dfrac{\sin x}{1+\cos x} = \csc x - \cot x$

conjugates $A+B$, $A-B$

DEN involves $1+\cos x$ ($1 \pm \cos x$, $1 \pm \sin x$)

$$\frac{\sin x}{(1+\cos x)} \cdot \frac{(1-\cos x)}{(1-\cos x)} = \frac{\sin x(1-\cos x)}{1-\cos x+\cos x-\cos^2 x}$$

$$= \frac{\sin x(1-\cos x)}{1-\cos^2 x}$$

$$= \frac{\cancel{\sin x}\cdot(1-\cos x)}{\sin^{\cancel{2}} x}$$

$$= \frac{1-\cos x}{\sin x}$$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \csc x - \cot x$$

☹

f) $\dfrac{2\sin x\cot x+\sin x-4\cot x-2}{2\cot x+1}=\sin x-2$

$$\frac{2\sin x\cot x+\sin x-4\cot x-2\cdot 1}{2\cot x+1}=\frac{\sin x(2\cot x+1)-2(2\cot x+1)}{2\cot x+1}$$

$$=\frac{(\sin x-2)\cancel{(2\cot x+1)}}{\cancel{2\cot x+1}}$$

$$=\sin x-2$$

g) $\dfrac{\sin^3 x+1}{\sin x+1}=\sin^2 x-\sin x+1$

$A^3+B^3=(A+B)(A^2-AB+B^2)$

$A=\sin x,\ B=1$

$$\frac{\sin^3 x+1^3}{\sin x+1}=\frac{\cancel{(\sin x+1)}(\sin^2 x-\sin x+1)}{\cancel{(\sin x+1)}}$$

$$=\sin^2 x-\sin x+1$$

Done

Identity Verification Guidelines

1. Work with the side with more "stuff."
2. Perform operations (+, -, x, squaring) or factor.
3. Use established identities.
4. Change to sines and cosines.
5. Mulitply by a special form of 1 (e.g. multiply a numerator and denominator by a conjugate).
6. Look at the other side of the equal sign to see if you are headed in the right direction.

Note: These guidelines can be helpful but they are not written in stone, so be flexible. Sometimes, for example, you will work with the side with less "stuff."