Sum, Difference, and Cofunctions Identities

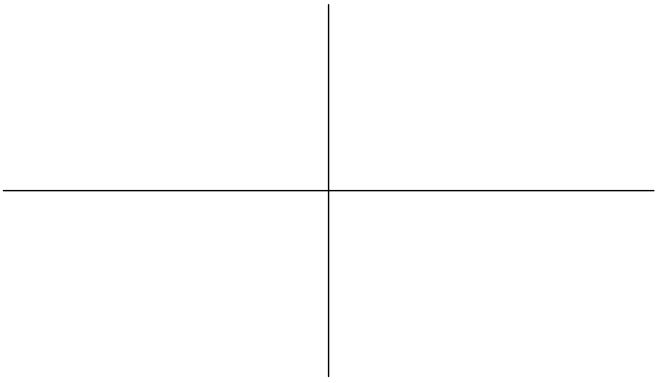
① $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$

② $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

③ $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$y = f(x)$ odd

$f(-x) = -f(x)$



(ex) Find exact value

$\sin(330° + 45°)$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \sin 330° \cos 45° + \sin 45° \cos 330°$ (ref $\angle$ 30°)

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

S	A
T	C

(ex) Find the exact value

$\cos\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{4}\right)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos\left(\frac{\pi}{12} + \frac{\pi}{4}\right)$

$\frac{\pi}{12} + \frac{\pi}{4}\cdot\frac{3}{3}$

$\frac{\pi}{12} + \frac{3\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$

$= \cos\left(\frac{\pi}{3}\right)$

$= \frac{1}{2}$

(ex) Given: $\sin\alpha = \frac{-7}{25}$, α is in Q IV

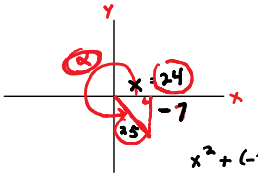
$\cos\beta = \frac{8}{17}$ (adj / hyp), β in Q II

Find $\sin(\alpha-\beta)$

$= \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$= \left(\frac{-7}{25}\right)\left(\frac{8}{17}\right) - \left(\frac{-15}{17}\right)\left(\frac{24}{25}\right)$

$= \frac{304}{425}$

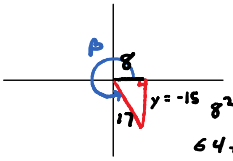


$x^2 + (-7)^2 = 25^2$

$x^2 + 49 = 625$

$x^2 = 576$

$x = 24$



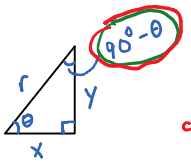
$8^2 + y^2 = 17^2$

$64 + y^2 = 289$

$\sqrt{y^2} = \sqrt{225}$

$y = -15$

Cofunction Identities



$\cos\theta = \frac{x}{r} = \sin(90^\circ-\theta)$

(1) $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$ (90° = $\frac{\pi}{2}$)

$\cos(90^\circ-\theta) = \sin\theta$

$\tan(90^\circ-\theta) = \cot\theta$

(ex) Rewrite in terms of sine: $\cos 80^\circ$

$\cos 80^\circ = \sin(90^\circ-80^\circ)$

$= \sin 10^\circ$

(ex) Verify

a) $\sin\left(\theta+\frac{\pi}{2}\right) = \cos\theta$

$\sin\left(\theta+\frac{\pi}{2}\right) = \sin\theta\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\theta$

$= \sin\theta\cdot 0 + (1)\cdot\cos\theta$

$= \cos\theta$

Done

b) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\tan(2\theta) = \tan(\theta+\theta)$ ($\alpha = \theta$, $\beta = \theta$)

$= \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$

$= \frac{2\tan\theta}{1-\tan^2\theta}$

Done

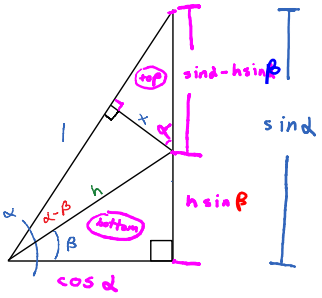
Double angle Identity

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\cos\alpha = ?$ (top triangle)

$\cos\beta = ?$ (bottom triangle)

$\sin(\alpha-\beta) = ?$



HW 6.2: 1-77 eo.l

8th ed.