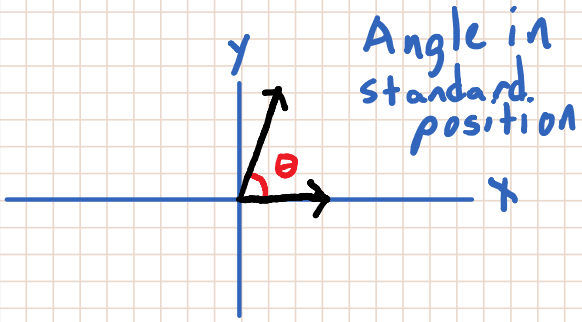
Section 5.3: Trigonometric Functions of Any Angle

Tuesday, January 21, 2014 12:24 PM

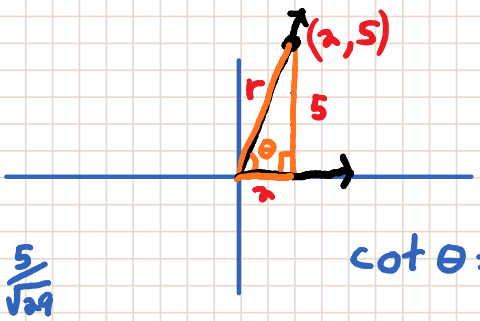
Goal: To find the values of the 6 trig ratios (functions) of any angle (not just acute).



Angle in standard position

Def: Any point except (0,0) can indicate an angle in std. position

(ex) a) Find the six trig fctns of the angle determined (2,5).



$r^2 = 2^2 + 5^2$

$r^2 = 29$

$r = \sqrt{29}$

$\sin\theta = \frac{5}{\sqrt{29}}$

$\cos\theta = \frac{2}{\sqrt{29}}$

$\tan\theta = \frac{5}{2}$

$\cot\theta = \frac{2}{5}$

$\sec\theta = \frac{\sqrt{29}}{2}$

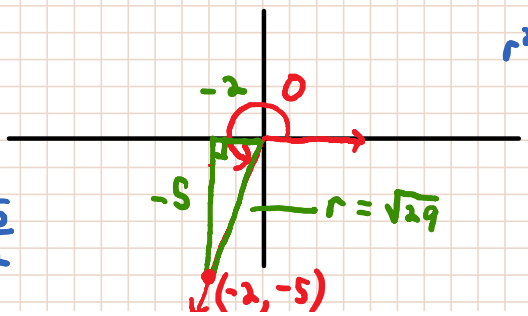
$\csc\theta = \frac{\sqrt{29}}{5}$

b) $(-2,-5)$ (same thing)

$r^2 = (-2)^2 + (-5)^2$

$r^2 = 29$

$r = \sqrt{29}$



$\sin\theta = \frac{-5}{\sqrt{29}}$

$\cos\theta = \frac{-2}{\sqrt{29}}$

$\tan\theta = \frac{-5}{-2} = \frac{5}{2}$

$\tan\theta = \frac{-2}{-2} = \frac{2}{2}$

$\cot\theta = \frac{2}{5}$

$\sec\theta = -\frac{\sqrt{29}}{2}$

$\csc\theta = -\sqrt{29}/5$

(-2, -5)

$90^\circ = \frac{\pi}{2} = 0.5\pi$

$180^\circ = \pi$ $\quad$ $0^\circ = 0$

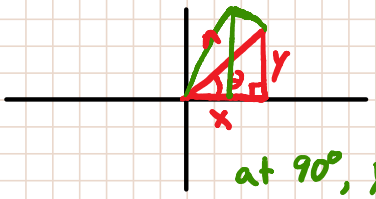
$270^\circ = \frac{3\pi}{2} = 1.5\pi$

Trig at the corners

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0 = 0^\circ$	0	1	0
$\pi/2 = 90^\circ$	1	0	ϕ
$\pi = 180^\circ$	0	-1	0
$\frac{3\pi}{2} = 270^\circ$	-1	0	ϕ

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

quiz



$\sin\theta = \frac{y}{r}$

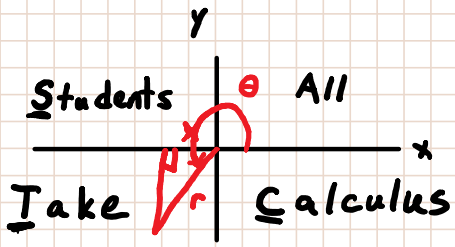
at 90°, $y = r$

so $\sin 90^\circ = \frac{y}{r} = \frac{r}{r} = 1$

ex Given: $\tan\theta = 1$ and $\sin\theta < 0$.

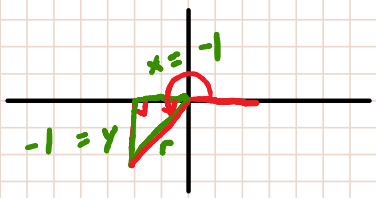
Find $\cos\theta$.

Find θ's quadrant



θ is in QIII

$\cos\theta = \frac{x}{r} = \frac{neg}{pos} = neg$



$\tan\theta = \frac{1}{1}$

$r^2 = (-1)^2 + (-1)^2$ $\quad$ $\cos\theta = \frac{-1}{r} \cdot \frac{\sqrt{}}{\sqrt{}}$

$r^2 = (-1)^2 + (-1)^2$

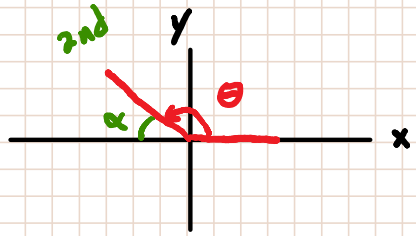
$r^2 = 2$

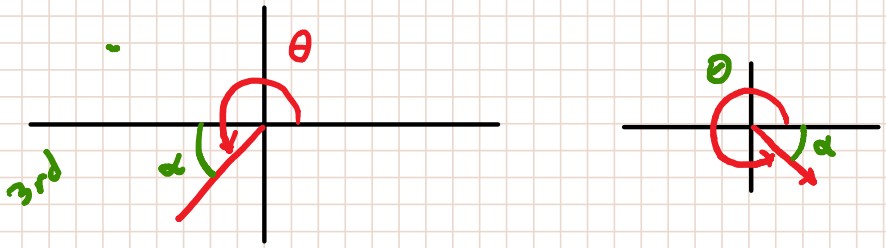
$r = \sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Reference Angles

The reference angle, α, of an angle θ in std. position is the acute angle formed between the terminal side of θ and the x-axis



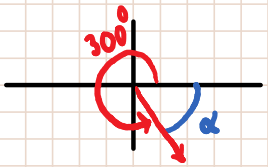


(ex) Find α.

a) 135°

$\alpha = 180^\circ - 135^\circ = 45^\circ$

b) 300°



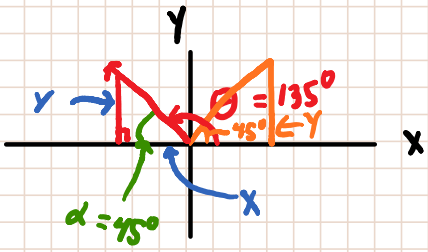
$\alpha = 360^\circ - 300^\circ = 60^\circ$

Big Idea: For a trig fctn, $f(\theta)$, $f(\theta) = \pm f(\alpha)$ (one or the other)

not both.

(ex) Find using ref. angles

a) $\sin 135^\circ$

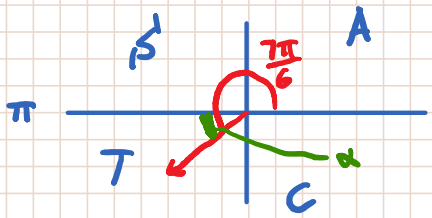


$$\sin 135^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

b) $\cos 135^\circ$

$$\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

c) $\tan \frac{7\pi}{6}$



$$\alpha = \frac{7\pi}{6} - \frac{6\pi}{6} = \pi/6$$

$$\tan \frac{7\pi}{6} = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$