Section 7.4 Part I: Complex Numbers

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2:04 PM

<u>Goal:</u> To $+, -, \cdot, \div$ complex numbers

solve: $x^2 = -1$

We invent the number $i = \sqrt{-1}$
to solve this equation.

$$i = \sqrt{-1}$$
$$i^0 = 1$$
$$i = i$$
$$i^2 = -1$$
$$i^3 = i^2 \cdot i = -i$$
$$i^4 = i^2 \cdot i^2 = 1$$

unique powers of i

$$i^5 = i^4 \cdot i = i$$
$$i^6 = -1$$
$$i^7 = -i$$
$$i^8 = i^4 \cdot i^4 = 1$$

ex simplify i^{99}

$$\begin{array}{r|l} & 24 \\ \hline 4 & 99 \\ & -8 \\ \hline & 19 \\ & -16 \\ \hline & R\ 3 \end{array}$$

← reduced exponent

$$i^{99} = i^3 = -i$$

ex simplify

a) $\sqrt{-25}$

$5i$

b) $\sqrt{-8}$

$= i\sqrt{8}$

$= i\,2\sqrt{2}$

$= 2i\sqrt{2}$

Any # of the form $a+bi$, where a and b are real is complex

ex $2+3i$

$a=2$, $b=3$

ex Perform the ops

a) $(1-2i)+(-3-4i)$

$-2-6i$

b) $(1-2i)-(-3-4i)$

$1-2i+3+4i$

$4+2i$

c) $(-3+5i)(4-6i)$

$-12 + \underbrace{18i + 20i} - 30i^2$

$-12 + 38i - 30(-1)$

$18 + 38i$

conjugates

$A+B$

$A-B$

d) $\dfrac{2}{(3-4i)} \cdot \dfrac{(3+4i)}{(3+4i)}$

$$\frac{6+8i}{9-16i^2}$$

$$\frac{6+8i}{25}$$

$$\frac{6}{25} + \frac{8}{25}i$$

e) $\sqrt{-2} \cdot \sqrt{-10}$

$$= \overbrace{i\sqrt{2} \cdot i}\sqrt{10}$$

$$= i^2\sqrt{20}$$

$$= -\sqrt{20}$$

$$= -2\sqrt{5}$$