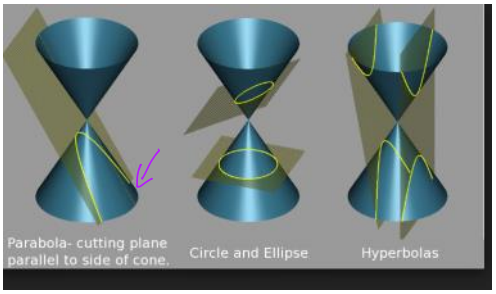
Parabolas and Ellipses (and Hyperbolas)

Goals:

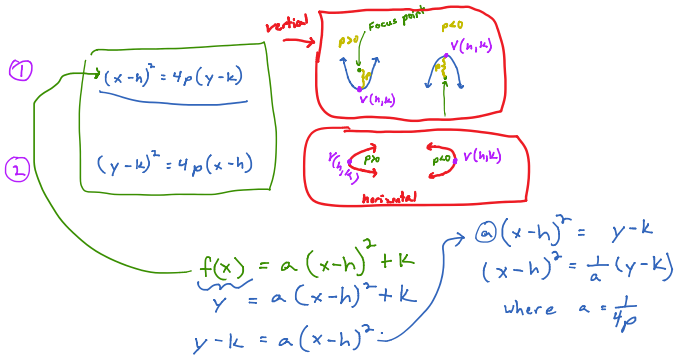
1. To graph these conics.
2. To write a given conic in standard form



Homework

1. See Announcement posted on Webassign
2. Watch YouTube Video for Section 8.5. Expect a quiz.

Standard Form of a Parabola with vertex (h,k)



Example: Find the vertex, axis of symmetry and graph the parabola.

a) $(x-2)^2 = 4(y+3)$

V(2,-3)
axis: x=2
4p=4
p=1>0 ∪

x	y
0	-2
4	-2

$4 = 4(y+3)$
$1 = y+3$
$y = -2$

$(x-h)^2 = 4p(y-k)$

a.o.s. x=2

$(y-k)^2 = 4p(x-h)$

b) $2y^2+3x+8y-4=0$

$-3x \quad +4 \quad -3x+4$

$2y^2+8y = -3x+4$

$+8$

$2(y^2+4y+4) = -3x+4+8$

$2(y+2)^2 = -3x+12$

$2(y+2)^2 = -\frac{3}{2}(x-4)$

a.o.s. $y=-2$

x	y
$1\frac{1}{3} = \frac{4}{3}$	0
$3\frac{1}{3}$	-1
$1\frac{1}{3}$	-4

$y=0$ 1st eqns

std. form

$y=-1$

$-\frac{2}{3} \; 1\frac{-2}{-3} -\frac{3}{2}(x-4)$

$-\frac{2}{3} = x-4$

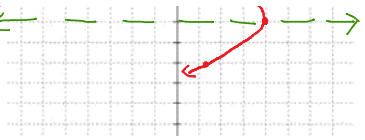
$x = 4 - \frac{2}{3}$

$\frac{12}{3} - \frac{2}{3}$

$2(y+2)^2 = -\frac{3}{2}(x-4)$

$\frac{1}{4}(y+2)^2 = -\frac{3}{2}(x-4)$

$V(4,-2)$ $\quad \frac{1}{4}4p = -\frac{3}{2}\cdot\frac{1}{4} \rightarrow p = -\frac{3}{8} < 0$



$y=0$ 1st eqns

$3x-4=0$

$x=\frac{4}{3}$

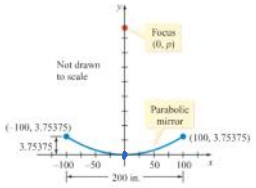
$x = 4 - \frac{2}{3}$

$\frac{12}{3} - \frac{2}{3}$

$\frac{10}{3}$

Reflective Property of a Parabola

48. **The Hale Telescope** The parabolic mirror in the Hale Telescope at the Palomar Observatory in Southern California has a diameter of 200 inches and a concave depth of 3.75375 inches. Determine the location of its focus (to the nearest inch).



Cross section of the mirror in the Hale Telescope

$(x-h)^2 = 4p(y-k)$

$x^2 = 4py$

$\frac{(100)^2}{4(3.75375)} = \frac{4p(3.75375)}{4(3.75375)}$

$p = \frac{100^2}{(4*3.753375)}$

$p \approx 666$ inches

$x^2 = (4.666)y$

Standard Equation of a Circle

$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = \frac{r^2}{r^2}$ [center (h,k) radius $= r$]

$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$

Standard Equations of an Ellipse with center (h,k)

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ major axis horizontal

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ major axis vertical

$a > b$

Example: Find the standard equation of the following ellipse and graph it.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

group like powers

$9x^2+4y^2-36x+24y+36=0$

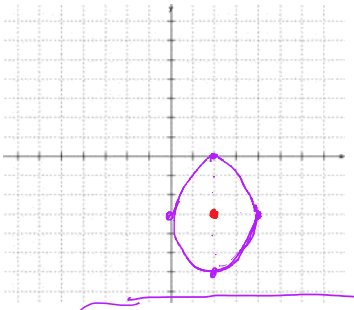
$9x^2-36x+4y^2+24y+36=0$

$9x^2-36x+4y^2+24y=-36$

$9(x^2-4x+4)+4(y^2+6y+9)=-36+36+36$

$\frac{9(x-2)^2}{36} + \frac{4(y+3)^2}{36} = \frac{36}{36}$

$\frac{(x-2)^2}{36/9} + \frac{(y+3)^2}{36/4} = 1$



$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

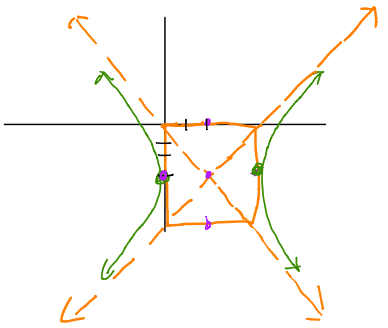
$C(2,-3)$

$\sqrt{4}=2$ left/right

$\sqrt{9}=3$ up/down 3

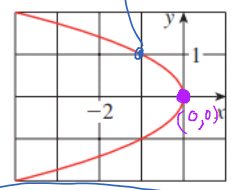
ex $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{9} = 1$

$C(2,-3)$ hyperbola



9–10 ■ Find an equation of the parabola. ~~Then find the focus and directrix.~~

9.



$(-1,1)$

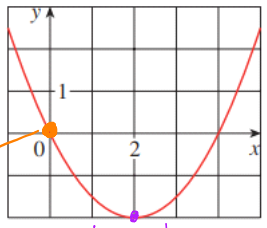
$(y-k)^2 = 4p(x-h)$

$y^2 = 4px$

$1^2 = 4p(-1) \rightarrow -4p = 1$

$p = -\frac{1}{4}$

10.



$(0,0)$

$(2,-2)$

$y^2 = 4\left(-\frac{1}{4}\right)x$

$y^2 = -x$

$(x-h)^2 = 4p(y-k)$

$(y-k)^2 = 4p(x-h)$

$(x-h)^2 = 4p(y-k)$

$(x-2)^2 = 4p(y+2)$

$4 = 4p(2)$

$p = \frac{1}{2}$

$(x-2)^2 = 2(y+2)$