Section 7.1: Gaussian Elimination

Wednesday, April 09, 2014
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Goal: To use matrices to solve systems of equations.

Definition: A matrix is a rectangular array of numbers. Rows run horizontally and columns run vertically.

ex

	Test 1	Test 2	Test 3	Test 4
Math 110	74	82	68	75
Math 115	75	66	71	73

$$A = \begin{bmatrix} 74 & 82 & 68 & 75 \\ 75 & 66 & 71 & 73 \end{bmatrix}$$

entry a_{14}

2 rows by 4 columns

2×4

order 2 rows by 4 columns

ex Given a system: $2x + 1y = 1$
$1x - 3y = 11$

coefficient matrix: $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$

constant matrix: $\begin{bmatrix} 1 \\ 11 \end{bmatrix}$

augmented matrix: $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -3 & 11 \end{bmatrix}$

Definition: An augmented matrix is in **row echelon form** (REF) if it satisfies:

1. All zero rows appear at the bottom of the matrix.
2. The first nonzero entry from in any row (looking left to right) is a 1 (called a leading 1).

3. For each nonzero row, the leading 1's appear in a stair-step pattern from (left to right) in subsequent rows.

ex

a) $\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & -6 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & -8 \\ 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

d) non example

$\begin{bmatrix} 1 & -2 & 3 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & -1 & 3 \end{bmatrix}$

ex solve the corresponding system

a) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$1x + 2y = 4$

$0x + 1y = 2$

$x + 2y = 4$

$y = 2$

$x + 2(2) = 4$

$x = 0$

b) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ 0 & 1 & -6 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$

$1x + 3y + 0z = -1$

$0x + 1y - 6z = 1$

$0x + 0y + 1z = -2$

$x + 3y = -1$

$y - 6z = 1$

$x = 0$

$(0, 2)$

$y - 6z = 1$

$z = -2$

$y + 12 = 1$

$y = -11$

$x + {}^{-}33 = -1$

$x = 32$

$(32, -11, -2)$

Row Operations

The following operations on an augmented matrix yield another augmented matrix whose corresponding system of equations has the same solutions as the original system

1. Switch any two rows.
2. Multiply a row by a constant.
3. Add multiples of rows together.

Note: We use row opps to convert an augmented matrix to REF and then solve the related system.

ex solve using Gaussian Elimination

a) $3x - 4y = 10$
$2x + 5y = -1$

$$\begin{array}{c} R_1 \\ R_2 \end{array} \left[\begin{array}{ccc} 3 & -4 & 10 \\ 2 & 5 & -1 \end{array}\right]$$

$$\begin{array}{c} R_1 \\ R_2 \end{array}\begin{bmatrix} 3 & -4 & 10 \\ 2 & 5 & -1 \end{bmatrix}_{2\times 3}$$

$R_1^* = \frac{1}{3}R_1$

$$\begin{array}{c} R_1 \\ R_2 \end{array}\begin{bmatrix} 1 & -\frac{4}{3} & \frac{10}{3} \\ 2 & 5 & -1 \end{bmatrix}$$

$$\begin{array}{ccc} -2 & \frac{8}{3} & \frac{-20}{3} \\ 2 & 5\ \frac{15}{3} & -1\ \frac{-3}{3} \\ \hline 0 & \frac{23}{3} & \frac{-23}{3} \end{array}$$

$R_2^* = -2R_1 + R_2$

$$\begin{array}{c} R_1 \\ R_2 \end{array}\begin{bmatrix} 1 & -\frac{4}{3} & \frac{10}{3} \\ 0 & \frac{23}{3} & \frac{-23}{3} \end{bmatrix}$$

$R_2^* = \frac{3}{23}R_2$

$$\begin{bmatrix} 1 & -\frac{4}{3} & \frac{10}{3} \\ 0 & 1 & -1 \end{bmatrix}$$

REF

Solve related system

$$x - \frac{4}{3}y = \frac{10}{3}$$

$$y = -1$$

$$x + \frac{4}{3} = \frac{10}{3}$$

$$x = \frac{6}{3} = 2$$

$$(2, -1)$$

b)
$$2x + y - 3z = 5$$
$$x - 2y + z = 0$$
$$3x - y - z = 5$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{cccc} 2 & 1 & -3 & 5 \\ 1 & -2 & 1 & 0 \\ 3 & -1 & -1 & 5 \end{array}\right]_{3\times 4}$$

write down augmented matrix

R_1^* = switch $R_1 \updownarrow R_2$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{cccc} 1 & -2 & 1 & 0 \\ 2 & 1 & -3 & 5 \\ 3 & -1 & -1 & 5 \end{array}\right]$$

$$R_2^* = -2R_1 + R_2$$
$$R_3^* = -3R_1 + R_3$$

$$\begin{array}{rrrrr} & -2 & 4 & -2 & 0 \\ & 2 & 1 & -3 & 5 \\ \hline R_2^* & 0 & 5 & -5 & 5 \end{array}$$

$$\begin{array}{rrrrr} & -3 & 6 & -3 & 0 \\ & 3 & -1 & -1 & 5 \\ \hline R_3^* & 0 & 5 & -4 & 5 \end{array}$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -5 & 5 \\ 0 & 5 & -4 & 5 \end{bmatrix}$$

$$R_2^* = \frac{1}{5} R_2$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & -4 & 5 \end{bmatrix}$$

$$\begin{array}{ccccc} & 0 & -5 & 5 & -5 \\ & 0 & 5 & -4 & 5 \\ \hline R_3^* & 0 & 0 & 1 & 0 \end{array}$$

$$R_3^* = -5R_2 + R_3$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

REF

$$x - 2y + z = 0$$

$$y - z = 1$$

$$z = 0$$

$$y = 1$$

$$(2, 1, 0)$$

$$x - 2(1) + 0 = 0$$
$$x - 2 = 0 \longrightarrow x = 2$$

c) $x - 3y + z = 4$
$x + 5y - z = 2$
$-2x + 2y - z = -7$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & -3 & 1 & 4 \\ 1 & 5 & -1 & 2 \\ -2 & 2 & -1 & -7 \end{bmatrix}$$

$$\begin{array}{cccc} -1 & 3 & -1 & -4 \\ 1 & 5 & -1 & 2 \\ \hline 0 & 8 & -2 & -2 \end{array} \; R_2^*$$

$R_2^* = -R_1 + R_2$

$R_3^* = 2R_1 + R_3$

$$\begin{array}{cccc} 2 & -6 & 2 & 8 \\ -2 & 2 & -1 & -7 \\ \hline 0 & -4 & 1 & 1 \end{array} \; R_3^*$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & -3 & 1 & 4 \\ 0 & 8 & -2 & -2 \\ 0 & -4 & 1 & 1 \end{bmatrix}$$

$R_2^* = \frac{1}{8} R_2$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & -3 & 1 & 4 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} \\ 0 & -4 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{cccc} 0 & 4 & -1 & -1 \\ 0 & -4 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$R_3^* = 4R_2 + R_3$

$$\left[\begin{array}{cccc} 1 & -3 & 1 & 4 \\ 0 & 1 & -1/4 & -1/4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

dependent

REF

$$x - 3y + z = 4$$

$$y - \frac{1}{4}z = -\frac{1}{4}$$

$$y = \frac{1}{4}z - \frac{1}{4}$$

$$x - 3\left(\frac{1}{4}z - \frac{1}{4}\right) + z = 4$$

$$4\left(x - \frac{3}{4}z + \frac{3}{4} + z = 4\right)$$

$$4x - 3z + 3 + 4z = 16$$

$$4x + z + 3 = 16$$

$$4x = -z + 13$$

$$x = -\frac{1}{4}z + \frac{13}{4}$$

$$\left(\underbrace{-\frac{1}{4}z + \frac{13}{4}}_{x},\ \underbrace{\frac{1}{4}z - \frac{1}{4}}_{y},\ \underbrace{z}_{z}\right)$$

let $c = z$ in webassign

$$\left(-\frac{1}{4}c + \frac{13}{4},\ \frac{1}{4}c - \frac{1}{4},\ c\right)$$

Notes on Gaussian Elimination

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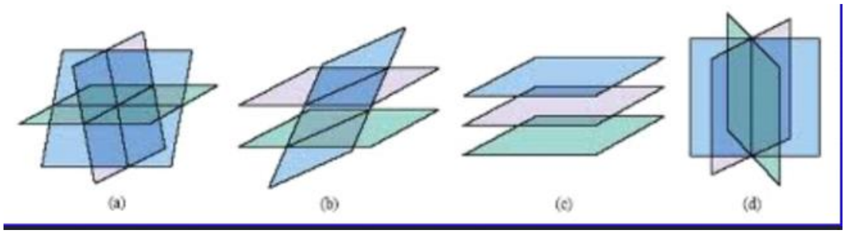
① A linear eqn. with 3 unknowns graphs to be a plane in space. Solving a linear system of 3 eqns, 3 unknowns means you find the intersection point(s) of all three planes.



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② If you get a row like 0 0 0 4, there is no solution (0=4? NO!)

③ If you get a row like 0 0 0 0, then the system is dependent.