Section 11.2: Arithmetic Sequences and Series

Monday, April 28, 2014
4:00 PM

Goals:

1. To find the nth term of an arithmetic sequence.
2. To find the sum of the first n terms of an arithmetic sequence.

Consider the sequence

$2, 7, 12, 17, \ldots 22, 27, \ldots$ arithmetic sequence

5 5 5

$a_n = 5n - 3$

From below formula $a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)5$

$a_n = 2 + 5n - 5$

$a_n = 5n - 3$

a linear function (domain is natural #'s)

recursive

This kind of sequence, $a_n = a_{n-1} + d$, is called arithmetic.

constant difference between each term

Def: In an arithmetic sequence each term is found by adding a common difference, d, to the previous term.

An Explicit Formula for a_n (arithmetic)

$$a_1 = a_1$$

$$a_2 = a_1 + 1d$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$$

$$a_4 = a_3 + d = (a_1 + 2d) + d = a_1 + 3d$$

$$a_5 = a_1 + 4d$$

$$\vdots$$

☆ $$\boxed{a_n = a_1 + (n-1)d}$$

☆ $\boxed{a_n = a_1 + (n-1)d}$

Explicit for a_n (for an arithmetic sequence)

(ex) Find a_n and a_{10} for

$2, 7, 12, 17, \ldots$

$a_n = 5n - 3$ (see above)

$a_{10} = 5(10) - 3$
$= 47$

(ex) Given $a_6 = -14$ and $a_8 = -20$,

Find a_7 and a_{15}.

$-14, \quad , \quad -20$

$a_6 \quad a_7 \quad a_8$

$n=6$

$d = \dfrac{-20 + (+14)}{2}$

$= \dfrac{-6}{2} = -3$

$a_7 = -14 - 3 = -17$

$\boxed{a_n = a_1 + (n-1)d}$

$n=6$

$-14 = a_1 + (6-1)(-3)$

a_6

$-14 = a_1 - 15$

$a_1 = 1$

$a_n = 1 + (n-1)(-3)$

$a_{15} = 1 + (15-1)(-3)$

$= 1 + (14)(-3)$

$= 1 + (-42)$

$= -41$

Def: The sum of terms of a sequence is called a series.

$$a_1, a_2, a_3, \ldots, a_n \Big\}\ \text{terms of sequence}$$

$$S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n \Big\}\ \text{series}$$

"sum of 1st n terms"

or "nth partial sum"

ex Find S_{100} for the counting numbers

$$\begin{aligned} 1S_{100} &= 1 + 2 + 3 + 4 + \cdots + 100 \\ +\quad 1S_{100} &= 100 + 99 + 98 + 97 + \cdots + 1 \end{aligned}$$

$$2S_{100} = \underbrace{101 + 101 + 101 + 101 + \cdots + 101}_{100 \text{ terms of } 101}$$

$$2S_{100} = 100(101)$$

$$S_{100} = 5050$$

For a general arithmetic series....

using a similar technique, the sum of generic arithmetic series is ...

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

ex Find the nth partial sum, S_n, of the sequence given by $a_n = 4n - 3$ where $n = 12$.

i.e. Find S_{12} (sum of 1st 12 terms)

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$a_1 = 4(1) - 3 = 1$, $a_{12} = 4(12) - 3$
$= 45$

$$S_{12} = \frac{12}{2}(1 + 45)$$
$$= 6(46)$$
$$= 276$$

ex Find S_n when $a_n = 4 - n$
and $n = 40$

$a_1 = 3$, $a_{40} = -36$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_4 = \frac{40}{2}(3 - 36)$$

$$= 20(-33)$$

$$= -660$$

ex

Theater Seating The seating section in a theater has 27 seats in the first row, 29 seats in the second row, and so on, increasing by 2 seats each row for a total of 10 rows. How many seats are in the 10th row, and how many seats are there in the section?

Aufmann, College Algebra, 8e
http://www.webassign.net/ebooks/aufcolalg8/shell.html?s=c90092d0840a7d7582721201ebef04ab&c=249283&f=4721409&type=youbook&id=
462
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$$27, \quad 29, \quad 31, \quad \ldots\ldots$$

seats 1st

2nd row

3rd row

$$a_n = a_1 + (n-1)d$$

$$a_n = 27 + (n-1)2$$

$$\left(\begin{array}{c}\text{\# of seats} \\ \text{in 10th row}\end{array}\right) = a_{10} = 27 + (10-1)(2)$$

$$= 27 + 18$$

$$= 45 \text{ seats}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$(n = 10)$

$$\left(\begin{array}{c}\text{\# Seats in} \\ \text{Section}\end{array}\right) = S_{10} = \frac{10}{2}(\underbrace{27}_{a_1} + \underbrace{45}_{a_{10}})$$

$$= 5(72)$$

$$= 360 \text{ seats}$$