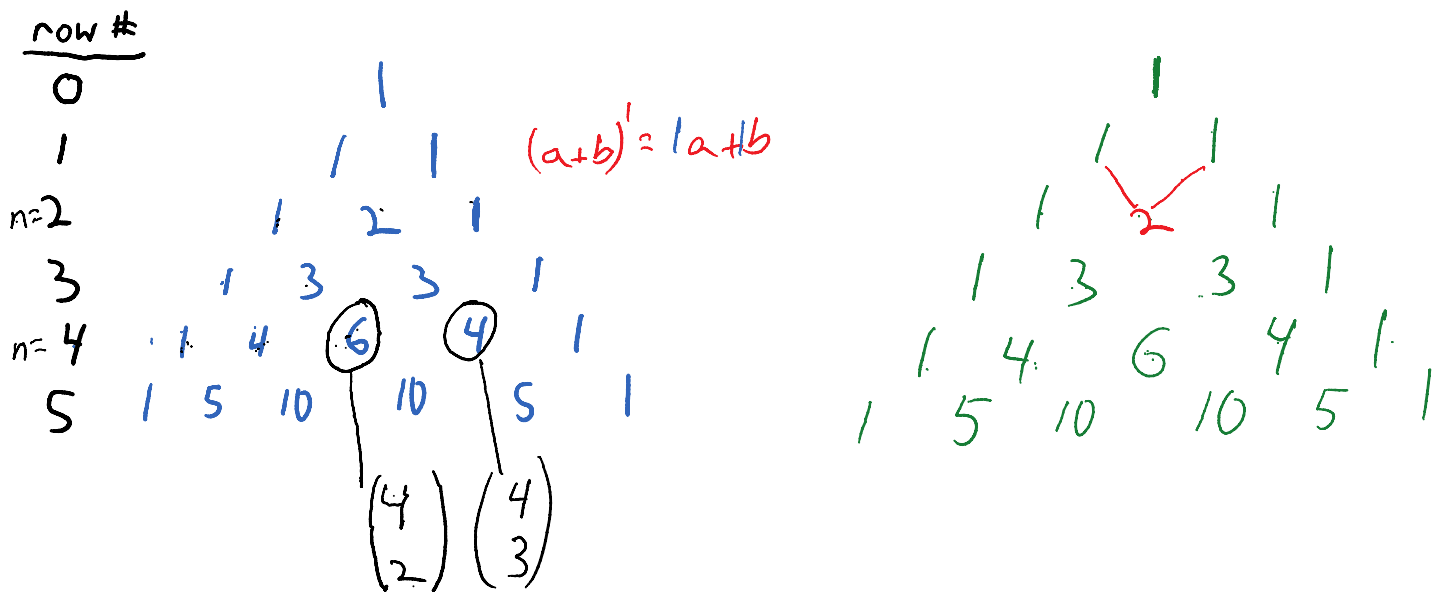


# Section 11.5: The Binomial Theorem

Wednesday, May 07, 2014  
5:02 PM

## Pascal's Triangle



$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2)$$

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$\underline{1a^3 + 3a^2b + 3ab^2 + 1b^3}$$

$$(a+b)^4 = \binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}b^4$$

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}b^n$$

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n} b^n$$

$\binom{n}{k} a^{n-k} b^k$

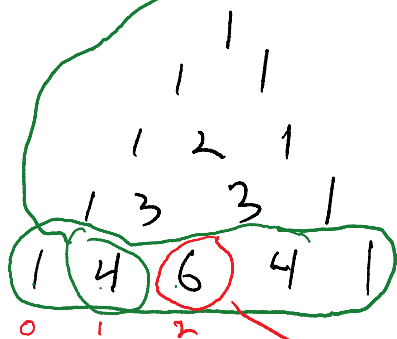
Notation:  $\binom{n}{k}$  "n choose k"

Stands for the binomial coefficient of  $(a+b)^n$  from the term in the <sup>binomial</sup> expansion whose variable part is  $a^{n-k} b^k$

ex) Compute  $\binom{4}{2}$

row # in pascals triangle

3rd # in the 4th row of P.T.



$$\binom{4}{2} = 6$$

$$b) \binom{3}{3} = 1 = \binom{3}{0} = \binom{n}{n} = \binom{n}{0}$$

## Binomial Theorem

$$\binom{n}{n-k} k \quad \binom{n}{n}$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{k} a^{n-k} b^k + \dots + \binom{n}{n} b^n$$

(ex) Find Fourth term:  $(x+2y)^{12}$

$\begin{matrix} a & b \\ \downarrow & \downarrow \\ (x+2y)^{12} \end{matrix}$

actual fourth term

$$\begin{aligned} \binom{12}{3} a^9 b^3 &= \binom{12}{3} a^9 b^3 \\ &= \binom{12}{3} x^9 (2y)^3 \\ &= 220 x^9 \cdot 8 y^3 \\ &= 1760 x^9 y^3 \end{aligned}$$

$\binom{n}{k}$   
 (k+1)st binomial coefficient

(ex) Find the term in the binomial expansion



$$\begin{aligned}(x-4y)^5 &= x^5 + 5x^4(-4y) + 10x^3(-4y)^2 + 10x^2(-4y)^3 + 5x(-4y)^4 + (-4y)^5 \\ &= x^5 - 20x^4y + 160x^3y^2 - 640x^2y^3 + 1280xy^4 - 1024y^5\end{aligned}$$