Section 11.5: The Binomial Theorem

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5:02 PM

Pascal's Triangle

row #	
0	1
1	1 1
n=2	1 2 1
3	1 3 3 1
n=4	1 4 6 4 1
5	1 5 10 10 5 1

$(a+b)^1 = 1a + 1b$

$$6 = \binom{4}{2} \qquad 4 = \binom{4}{3}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = (a+b)(a^2+2ab+b^2)$$
$$= a^3 + 2a^2b + ab^2 + b^3$$
$$\quad 1a^2b + 2ab^2$$
$$1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^{4\,(n)} = \binom{4}{0}a^4 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}b^4$$

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n}b^n$$

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n}b^n$$

$$\binom{n}{k}a^{n-k}b^{k}$$

Notation: $\binom{n}{k}$ "n choose k"

Stands for the binomial coefficient of $(a+b)^n$ from the term in the binomial expansion whose variable part is $\boxed{a^{n-k}b^{k}}$

ex Compute a) $\binom{4}{2}$

row # in pascals triangle

3rd # in from the 4th row of P.T.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
0 1 2

$$\binom{4}{2} = 6$$

b) $\binom{3}{3} = 1 = \binom{3}{0} = \binom{n}{n} = \binom{n}{0}$

Binomial Theorem

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{k}a^{n-k}b^k + \cdots \binom{n}{n}b^n$$

ex Find $\underbrace{\text{Fourth term}}_{\text{actual fourth term}}$: $(\overset{a}{x} + \overset{b}{2y})^{12}$

$$\binom{12}{3}a^{12-3}b^3 = \binom{12}{3}a^9b^3$$

$$= \binom{12}{3}x^9(2y)^3$$

$$= 220x^9 \cdot 8y^3$$

$$= 1760x^9y^3$$

$\binom{n}{k}$ $(k+1)$st binomial coefficient

ex Find the term in the binomial expansion

$(3r+2s)^9$ that contain s^7

$\underbrace{3r}_{3r} \quad \underbrace{2s}_{b}$

$n=9$

$$\binom{9}{7} a^{\boxed{2}} b^{7} = \binom{9}{7} a^2 b^7$$

$9-7$, k

$$= \binom{9}{7} (3r)^2 (2s)^7$$

$$= \binom{9}{7} 9r^2\, 128 s^7$$

$$= 41472 r^2 s^7$$

(ex) multiply out: $(x - 4y)^5$

$\underbrace{x}_{a} \quad \underbrace{-4y}_{b=-4y}$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$(a+b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$$

$16y^2$ $\quad -64y^3$ $\quad 256y^4$

$$(x-4y)^5 = x^5 + 5x^4(-4y) + 10x^3(-4y)^2 + 10x^2(-4y)^3 + 5x(-4y)^4 + (-4y)^5$$

$$= x^5 - 20x^4y + 160x^3y^2 - 640x^2y^3 + 1280xy^4 - 1024y^5$$