Section 3.1: The Remainder and Factor Theorems

Monday, October 13, 2014
5:04 PM

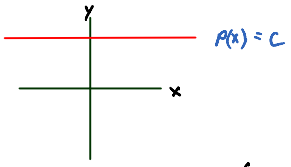
Def: A polynomial function has the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

examples

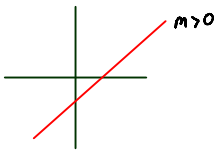
① $P(x) = \overset{a_0}{c}$, c a constant

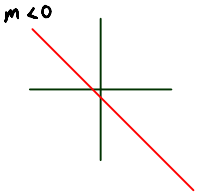
degree is 0



$P(x) = c$

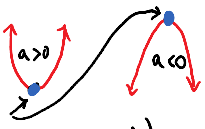
② $P(x) = \underset{a_1}{m}\, x + \underset{a_0}{b}$ (linear polynomial fctn.) deg = 1





③ $P(x) = ax^2 + bx + c$ $\left(P(x) = a_2 x^2 + a_1 x^1 + a_0\right)$

(quadratic poly deg = 2)



$$\left(\frac{-b}{2a}, \underbrace{P\left(\frac{-b}{2a}\right)}_{\text{max or min}}\right)$$

vertex formula

ex Long Divide

a) $\dfrac{x^2+4x-8}{x-3} = x + 7 + \dfrac{13}{x-3}$

$$\begin{array}{r|l} & x + 7 \\ \hline x-3 & x^2 + 4x - 8 \\ & -x^2 + 3x \\ \hline & 7x - 8 \\ & -7x + 21 \\ \hline & 13 \end{array}$$

Q: $x+7$

R: 13

check

dividend = (div)(Quo) + R

$(x-3)(x+7) + 13$

$x^2 + 4x - 21 + 13$

$x^2 + 4x - 8$ ✓

Synthetic Division

Works when the divisor is of the form $x - c$, c a constant.

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Works when the divisor is of the form $x-c$, c a constant.

ex Divide using syn. div.

a) $(x^3-x^2+2x-3)\div(x-1)$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 2 & -3 \\ & & 1 & 0 & 2 \\ \hline & 1 & 0 & 2 & \boxed{-1} \end{array}$$

Q: $1x^2+0x+2 = x^2+2$

R: -1

b) $\dfrac{z^2-z+2+z^4}{z-2} = \dfrac{1z^4+0z^3+1z^2-1z+2}{z-2}$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 1 & -1 & 2 \\ & & 2 & 4 & 10 & 18 \\ \hline & 1 & 2 & 5 & 9 & \boxed{20} \end{array}$$

Q: z^3+2z^2+5z+9

R: 20

The Remainder Theorem

If R is the remainder when P(x) is divided by x-c, then R=P(c).

ex Let $P(x)=x^3-3x^2+x+1$

a) Find the remainder when $P(x)$ is divided by $x+2$

$$\begin{array}{r|rrrr} -2 & 1 & -3 & 1 & 1 \\ & & -2 & 10 & -22 \\ \hline & 1 & -5 & 11 & \boxed{-21} \end{array}$$

$R=-21$

Q: $x^2-5x+11$

R: -21

check

$$\underbrace{x^3-3x^2+x+1}_{P(x)} = \underbrace{(x+2)(x^2-5x+11)}_{P(x)} \overbrace{-21}^{R}$$

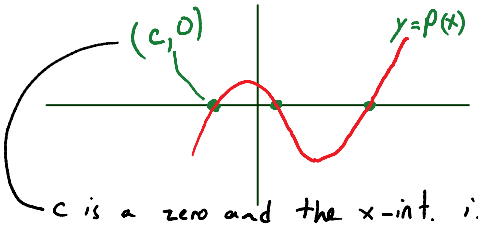
when $x=-2$, we get $P(-2)=-21$

b) Find $P(-2)$

$$\begin{aligned} P(-2) &= (-2)^3-3(-2)^2-2+1 \\ &= -8-20-2+1 \\ &= -21 \end{aligned}$$

Def: A number c is a zero of $P(x)$ iff $P(c)=0$

Note: Real zeros of $P(x)$ correspond to x-intercepts.



c is a zero and the x-int. is $(c, 0)$.

The Factor Theorem

The number c is a zero of a polynomial, P(x), iff x-c is a factor of P(x).

ex Find the zeros of
$P(x) = x^2 - x - 6$

$0 = x^2 - x - 6$

$0 = (x+2)(x-3)$

$x + 2 = 0$ or $x - 3 = 0$

$x = -2$ $\quad$ $x = 3$

zeros of $P(x)$

ex Let $P(x) = 3x^4 + 8x^3 + 10x^2 + 2x - 20$

a) show $c = -2$ is a zero of $P(x)$

-2	3	8	10	2	-20
		-6	-4	-12	20
	3	2	6	-10	0

$0 = P(-2) = R$ by Remainder Th.

So -2 is a zero of $P(x)$

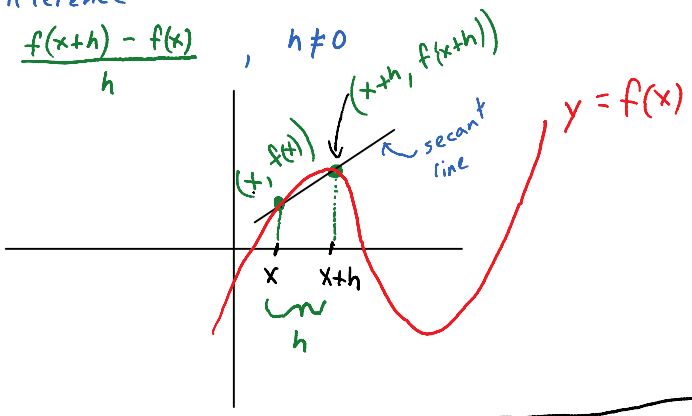
Q: $3x^3 + 2x^2 + 6x - 10$

Divisor $= x - (-2) = x + 2$

b) factor $P(x)$ into a linear factor times a cubic factor

$$3x^4 + 8x^3 + 10x^2 + 2x - 20 = \underbrace{(x+2)}_{\text{divisor}}\underbrace{(3x^3 + 2x^2 + 6x - 10)}_{\text{Quotient}}$$

Def: Suppose $f(x)$ is a function.
The difference quotient of f
is $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$



m between $(x, f(x))$ and $(x+h, f(x+h))$
is the D.Q.

$$D.Q. = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

ex Find the difference quotient:
$f(x) = x^2 + 5$

$$f(x+h) = (x+h)^2 + 5$$

$$D.Q. = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 + 5 - (x^2 + 5)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5} - \cancel{x^2} - \cancel{5}}{}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x+h$$

22. 0/7 points — AufCAT7 2.6.080. [2727026]

The distance traveled by a ball rolling down a ramp is given by $s(t) = 4t^2$, where t is the time in seconds after the ball is released and $s(t)$ is measured in feet. The ball travels 4 feet in 1 second and 16 feet in 2 seconds. Use the difference quotient for average velocity given below to evaluate the average velocity for each of the following time intervals.

$$\frac{s(a+\Delta t) - s(a)}{\Delta t}$$

(a) $[2, 3]$ (*Hint*: In this case, $a = 2$ and $\Delta t = 1$.)

ft/sec

Compare this result to the slope of the line through $(2, s(2))$ and $(3, s(3))$.

This is —Select— the slope of the line through $(2, s(2))$ and $(3, s(3))$.

(b) $[\overset{a}{2}, 2.5]$ $\Delta t = 2.5 - 2 = 0.5$

ft/s

(c) $[\overset{a}{2}, 2.1]$

ft/s

a) $\dfrac{s(3) - s(2)}{3-2}$ $\qquad s(t) = 4t^2$

$= \dfrac{36-16}{1}$

$= 20 \ \dfrac{ft}{sec}$

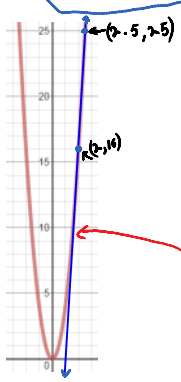
b) $\dfrac{s(2+0.5) - s(2)}{0.5} = \boxed{\dfrac{s(2.5) - s(2)}{2.5-2}}$ slope formula

2.5-2

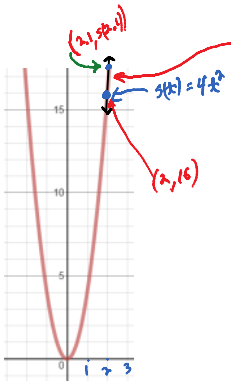
$= \dfrac{25-16}{0.5}$

$= \dfrac{9}{.5}$

$= 18 \ ft/sec$



Geometrically, in part (b), we found that the slope of the secant line (in blue) between points (2,16) and (2.5,25) is 18



part (c) you're finding the slope of this secant line