Goals:

1. To analyze the behavior of polynomials for extreme x-values.
2. To analyze the behavior of polynomial function at x-intercepts.
3. To find relative extrema.
4. To graph polynomial functions.

Leading Term Test: For extreme $\underbrace{\text{x-values}}_{|x| \text{ is "big"}}$ of a polynomial, as goes the leading term, so goes the entire polynomial.

Ex. Use the Leading Term Test to determine the far left and far right behavior of the function:

$$P(x) = -6x^4 + 2x^3 - 5x^2 + x - 1$$

leading term is $y = -6x^4$ ← both $y = -6x^4$ and $P(x)$ look same for extreme values of x

so as $x \to \infty$, $P(x) \to -\infty$

as $x \to -\infty$, $P(x) \to -\infty$

Ex. Determine the behavior of the graph of P(x) at the x-intercepts.

$$P(x) = x^4 (x+2)^3 (x-3)^2 (x+1)^5$$

$$0 = x^4 (x+2)^3 (x-3)^2 (x+1)^5$$

$x^4 = 0$, $x + 2 = 0$, $x - 3 = 0$, $x + 1 = 0$

$x = 0$, $x = -2$

$(0,0)$ $(-2,0)$ $(3,0)$ $(-1,0)$

$(0,0)$: associate factor has even exponent of 4. The graph does not cross through the x-axis

$(-2,0)$: exp $= 3$ (odd) graph crosses x-axis

$(3,0)$: doesn't cross x-axis

$(-1,0)$: crosses x-axis

Ex. Graph P(x). Be sure to. . .

1. Use LTT
2. Plot intercepts
3. Plot points
4. Use symmetry when applicable

a) $P(x) = 1x^3 - 6x^2 + 9x + 0$

① $y = x^3$

$x \to \infty,\ P(x) \to \infty$

$x \to -\infty,\ P(x) \to -\infty$

② $0 = x^3 - 6x^2 + 9x$

$0 = x(x^2 - 6x + 9)$

$0 = \underbrace{x^1 (x-3)^2}_{P(x)\ \text{factored}}$

$x = 0$	$x = 3$
$(0,0)$	$(3,0)$
exp = 1: passes through x-axis	exp = 2: doesn't cross x-axis

x-int

x-int

$P(0) = 0$

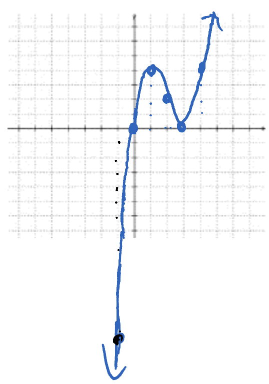
$(0,0)$

③

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 9 & 0 \\ & & -1 & 7 & -16 \\ \hline & 1 & -7 & 16 & \boxed{-16} \end{array}$$

$p(-1) = -16$

x	$p(x)$
-1	-16
1	4
2	2
4	4
1	4 rel max



b) $P(x) = -x^4 + 2x^3 + 3x^2 - 4x - 4$

$= -(x-2)^2 (x+1)^2$

① $y = -x^4$

$P(x) \to -\infty$

① $y = -x^4$

as $x \to \infty$, $P(x) \to -\infty$

as $x \to -\infty$, $P(x) \to -\infty$

② x-int

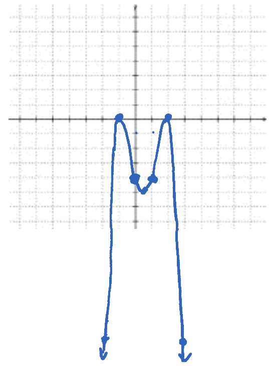
$(2,0)$ doesnt cross x-axis

$(-1,0)$ graph doesn't cross x-axis here

y-int

$P(0) = -4$

$(0,-4)$



③

x	y
-2	-16
1	-4
3	-16
0.5	-5.0625

relative minimum from calculator

c) $P(x) = x^3 - x$

① see (a)

② $0 = x^3 - x$

$0 = x(x^2 - 1)$

$0 = x^1(x+1)^1(x-1)^1$ by 1's

$x = 0, -1, 1$

$(0,0)$ $(-1,0)$ $(1,0)$ ← x-int

also y-int

~~pass~~ crosses x-axis

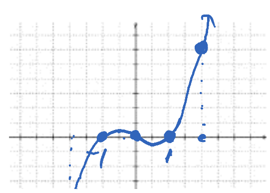
③

x	P(x)
2	6
.5	-0.375
-2	-6

$P(x) = x^3 - x$

$P(-x) = (-x)^3 - (-x)$

$= -x^3 + x$



by 2's

.5	-0.375
-2	-6
-.5	.375
≈ 0.58	-0.38
≈ -0.58	0.38

rel min

rel max

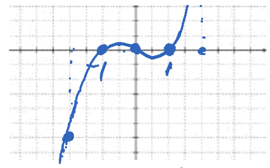
$P(-x) = (-x)^3 \ldots$

$= -x^3 + x$

$= -(x^3 - x)$

$P(x)$ odd

origin symmetry

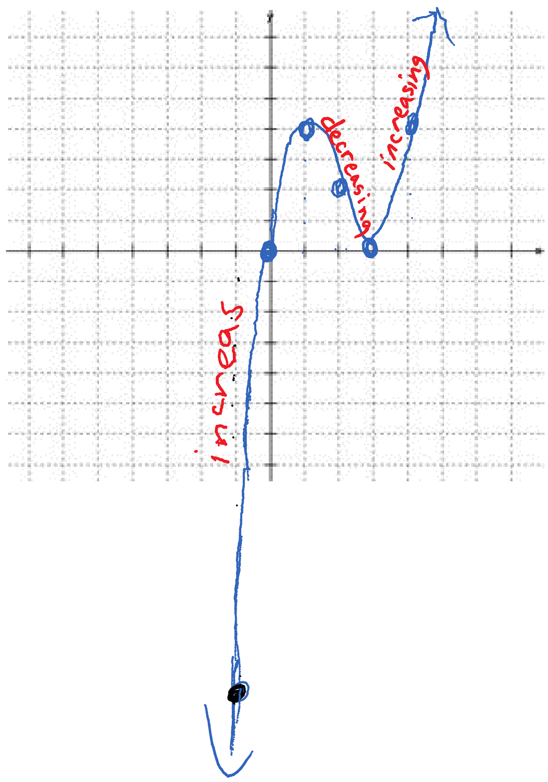


by 2's

by 1's

Ex. Find the intervals of increase/decrease from the function in part (a) of the last example. } go left to right

The graph from part (a) is . . .



These are the x-intervals for which $P(x)$ is increasing or decreasing

Increasing on $(-\infty, 1)$ and $(3, \infty)$

Decreasing on $(1, 3)$

Recall:

Function of the form...	Transforms the graph of y = f(x)...
y = f(x)+k	up k units
y = f(x)-k	down k units
y = f(x-h)	right h units
y = f(x+h)	left h units
y= Af(x)	by a vertical stretch/shrink factor of A
y = f(Bx)	by a horizontal stretch/shrink factor of 1/B
y = -f(x)	by a reflection across the x-axis