Section 3.3: The Zeros of a Polynomial

Wednesday, October 15, 2014
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Recall: ① The number c is a zero of $P(x)$ iff $P(c)=0$

② The real zeros of $P(x)$ correspond to x-intercepts

Def: The number c is a zero of multiplicity k iff $(x-c)^k$ is a factor of $P(x)$.

ex Find zeros of $P(x)$ and their multiplicities:

$$P(x) = x^2(x-3)^4(x+1)^5(x-2)^{10}$$

zeros: 0 , 3 , -1 , 2

mult: 2 4 5 10

The Rational Zero Theorem

If $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0$ is a polynomial with integer coefficients, and $\frac{p}{q}$ is a simplified rational zero of $P(x)$, then p is a factor of a_0 and q is a factor of a_n.

ex Find the zeros. State multiplicities.

a) $P(x) = 2x^3 + x^2 - 18x - 9$

factors of $a_0 = -9$: $\pm\{1, 3, 9\}$

factors of $a_n = a_3 = 2$: $\pm\{1, 2\}$

① Possible Rational zeros (PRZ's):

$$\pm\left\{1, \overset{-\frac{1}{2}}{\frac{1}{2}}, \overset{\pm 3}{3}, \frac{3}{2}, 9, \frac{9}{2}\right\}$$

② Get prime suspects using calculator

③ verify w/ syn div

$$\begin{array}{r|rrrr}
-3 & 2 & 1 & -18 & -9 \\
 & & -6 & 15 & 9 \\ \hline
3 & 2 & -5 & -3 & \boxed{0} \\
 & & 6 & 3 & \\ \hline
-\frac{1}{2} & 2 & 1 & \boxed{0} & \\
 & & -1 & & \\ \hline
 & 2 & \boxed{0} & &
\end{array}$$

$\leftarrow P(-3) = 0$ (Remainder Th)

zeros are $\pm 3, -\frac{1}{2}$ multiplicities are all 1

Factor $P(x) = 2(x+3)(x-3)(x+\frac{1}{2})$

b) $P(x) = 1x^3 - 2x + 1$

① $\begin{cases} \text{factors of } a_0 = 1 : \pm 1 \\ \text{factors of } a_3 = 1 : \pm 1 \end{cases}$

PRZ's : ± 1

② ✓

③
$$\begin{array}{r|rrrr}
1 & 1 & 0 & -2 & 1 \\
 & & 1 & 1 & -1 \\ \hline
 & 1 & 1 & -1 & \boxed{0}
\end{array}$$

$Q: \; 1x^2 + 1x - 1 = x^2 + x - 1$

$1x^2 + 1x - 1 = 0$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

zeros are $1, \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$

all have multiplicity 1

c) $P(x) = 4x^4 - 4x^3 + 13x^2 - 12x + 3$

factors of 3: $\pm\{1, 3\}$

" " 4: $\pm\{1, 2, 4\}$

PRZ's: $\pm\{1, \frac{1}{2}, \frac{1}{4}, 3, \frac{3}{2}, \frac{3}{4}\}$

$\frac{1}{2}$	4	-4	13	-12	3
		2	-1	6	-3
$\frac{1}{2}$	4	-2	12	-6	0
		2	0	6	
	4	0	12	0	

$4x^2 + 12 = 0$

$x^2 = -3$

$x = \pm\sqrt{3}$

$$\underbrace{x = \pm i\sqrt{3}, \; \frac{1}{2} \; (\text{mult } 2)}_{\text{zeros.}}$$

ex

19. 0/4 points

Evaluate the piecewise-defined function for the indicated values.

$$Q(t) = \begin{cases} 4, & \text{if } 0 \le t \le 5 \\ -t + 7, & \text{if } 5 < t \le 8 \\ \sqrt{t-7}, & \text{if } 8 < t \le 11 \end{cases}$$

(a) $Q(0) =$ $\boxed{4}$ ✗

(b) $Q(x)$, $6 < x < 7 =$ $\boxed{-x+7}$ ✗ since x also lies between 5 and 8

(c) $Q(n)$, $1 < n < 2 =$ $\boxed{4}$ ✗

(d) $Q(m^2 + 7)$, $1 < m \le 2 =$ $\boxed{m}$ ✗ since $8 < m^2 + 7 \le 11$ when $1 < m \le 2$

$\left(\sqrt{(m^2+7)-7} = \sqrt{m^2} = m \right)$

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