Goal: To find the zeros of a polynomial

$n \geq 1$

Theorem (FTA): Any polynomial of degree n with complex coefficients has exactly n zeros, counting multiplicities.

, a) Find the zeros:

$P(x) = 1x^4 - 3x^2 - 4$

Factors of -4: $\pm\{1, 2, 4\}$

actors of 1: ± 1

RZ's: $\pm\{1, 2, 4\}$ ← suspects

(e expect $n = 4$ zeros by FTA)

2	1	0	-3	0	-4
		2	4	2	4
-2	1	2	1	2	0
		-2	0	-2	
	1	0	1	0	

$x^2 + 1 = 0$

quotient contains remaining zeros

$\sqrt{x^2} = \pm\sqrt{-1}$

$x = \pm i$

$x = \pm i$

zeros : $x = \pm 2, \pm i$

actor $P(x)$: $P(x) = (x-2)(x+2)(x-i)(x+i)$

6 $P(t) = 4t^3 - 10t^2 + 4t + 5$

factors of 5: $\pm\{1, 5\}$

" " 4: $\pm\{1, 2, 4\}$

PRZ's : $\pm\left\{1, \frac{1}{2}, \frac{1}{4}, 5, \frac{5}{2}, \frac{5}{4}\right\}$ $\frac{\cancel{p}}{\cancel{q}}$

There are 3 zeros, by FTA)

4	-10	4	5
	-2	6	-5
4	-12	10	0

$\frac{4t^2}{2} - \frac{12t}{2} + \frac{10}{2} = \frac{0}{2}$

$2t^2 - 6t + 5 = 0$

$t = \frac{6 \pm \sqrt{36 - 4(2)(5)}}{4}$

$t = \frac{6 \pm \sqrt{-4}}{4}$

$t = \frac{6 \pm 2i}{4}$

$$t = \frac{\cancel{2}(3\pm i)}{\cancel{4}}$$

$$t = \frac{3\pm i}{2}, \quad -\frac{1}{2}$$

zeros: $\underbrace{\frac{3}{2}-\frac{1}{2}i,\ \frac{3}{2}+\frac{1}{2}i}_{\text{complex conjugates}},\ -\frac{1}{2}$

$$P(t) = 4\left(t+\frac{1}{2}\right)\left(t-\left(\frac{3}{2}-\frac{1}{2}i\right)\right)\left(t-\left(\frac{3}{2}+\frac{1}{2}i\right)\right)$$

Conjugate Pairs Theorem

If $a+bi$ is a zero of $P(x)$, and $P(x)$ has real coefficients, then $a-bi$ is also zero.

STUDY THIS

ex) Let $P(x) = x^4 - 2x^3 - x^2 + 6x - 6$.

Given $1-i$ is a zero of $P(x)$, find the rest.

$1-i$	1	$-2+0i$	-1	6	-6
		$1-i$	-2	$-3+3i$	6
	1	$-1-i$	-3	$3+3i$	$\boxed{0}$
$1+i$		$1+i$	0	$-3-3i$	

$$\begin{array}{cccc} 1 & 0 & -3 & \boxed{0} \end{array}$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

Zeros: $1 \pm i, \ \pm\sqrt{3}$

ex) Given $1+2i$ and 3 are zeros of a cubic polynomial w/ real coefficients, find an equation for that polynomial.

Since the coefficients are real, by CPT, $1-2i$ is also a zero.

Zeros: $1+2i, \ 1-2i, \ 3$

$$P(x) = (x-3)[x-(1+2i)][x-(1-2i)]$$

$$= (x-3)\left[x^2 - x(1-2i) - x(1+2i) + (1+2i)(1-2i)\right]$$

$$= (x-3)\left(x^2 - x + \cancel{2ix} - x - \cancel{2ix} + 1 - 4\overset{+4}{i^2}\right)$$

$$= (x-3)(x^2 - 2x + 5)$$

$$= x^3 - 2x^2 + 5x - 15$$

$$\boxed{P(x) = x^3 \overset{-3x^2}{-5x^2} \overset{+6x}{+11x} - 15}$$

How would you find the zeros of

$x^5 + 32 = 0$?

$$\sqrt[5]{x^5} = \sqrt[5]{-32}$$

$$x = -2$$

<u>Time-out</u> (Back to trig land)

$$z = a + bi = r(\cos\theta + i\sin\theta)$$

$$= \underbrace{r \operatorname{cis}\theta}_{\text{trig form of } z,\ \text{where } r = |z|}$$

<u>Demoivre</u>

$$z^n = r^n \operatorname{cis}(n\theta)$$

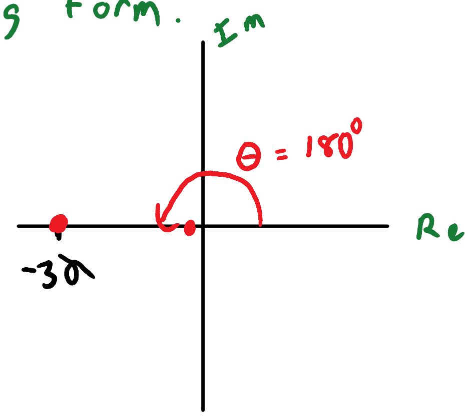
Recall: $2^{\frac{1}{4}} = \sqrt[4]{2}$, $a^{\frac{1}{n}} = \sqrt[n]{a}$

$$\boxed{z^{\frac{1}{n}} = r^{\frac{1}{n}} \operatorname{cis}\left(\frac{\theta}{n}\right)}$$

ex Find all five zeros of $x^{5} + 32 = 0$ (n)

(i.e. find the five fifth roots of -32)

Write -32 in trig form.



$z = -32 + 0i$

$z = r \operatorname{cis} \theta$

$r = 32$

$z = 32 \operatorname{cis} 180°$

$z^n = r^n \operatorname{cis}(n\theta)$

$z^{\frac{1}{n}} = r^{\frac{1}{n}} \operatorname{cis}\left(\frac{\theta}{n}\right)$

$$w_k = r^{\frac{1}{n}} \operatorname{cis}\left(\overbrace{\frac{\theta + 360°k}{n}}^{\frac{\theta}{n} + \frac{360° k}{n}}\right)$$

$k = 0, 1, 2, \ldots, n-1$

$z^{\frac{1}{5}} = 32^{\frac{1}{5}} \operatorname{cis} \frac{180°}{5} = 2 \operatorname{cis} 36°$

In the lingo from above what I just found is ....

$\frac{360°}{n} = \frac{360°}{5} = 72°$

$w_0 = 2 \operatorname{cis} 36°$

$w_1 = 2 \operatorname{cis} (36° + 72°) = 2 \operatorname{cis} 108°$

$w_2 = 2 \operatorname{cis} 180°$

$w_2 = 2 \text{ cis } 180^\circ$

$w_3 = 2 \text{ cis } 252^\circ$

$w_4 = 2 \text{ cis } 324^\circ$

keep adding 72° to get other zeros (roots)

These are the zeros of $x^5 + 32 = 0$