- Section 3.5: Graphing Rational Functions

Monday, February 10, 2014
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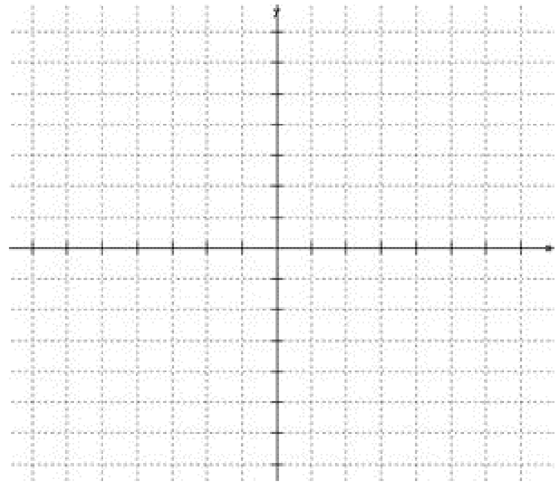


Def: A rational function is a ratio of two polynomial functions.

$$R(x) = \frac{P(x)}{Q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \dots + b_1 x + b_0}, \quad Q(x) \neq 0$$

ex
$$R(x) = \frac{x^2+7x+2}{x-1}$$

$$R(x) = \frac{3}{2} = \frac{a_0}{b_0}$$

Asymptotes - A line that a graph gets and close to but never quite touches

3 kinds: (1) vertical, (2) horizontal, (3) slant

(1) Vertical asymptotes (V.A.)

- can occur where a $R(x)$ is undefined. (when $Q(x) = 0$)
- find by setting DEN = 0 and solve.

ex Find VA's.

$$R(x) = \frac{x+1}{x^2+x-2}$$

$$1x^2 + 1x - 2 = 0$$

$$(x-1)(x+2) = 0$$

$$x = 1 \quad \text{or} \quad x = -2$$ eqns. of V.A.'s

Find V.A.'s

ex a) $f(x) = \dfrac{x}{x^2+1}$

$$x^2 + 1 = 0$$

$$\sqrt{x^2} = \pm\sqrt{-1}$$

$x = \pm i \rightarrow$ no V.A.'s (No real x-values that make DEN = 0)

b) $g(x) = \dfrac{x^2+7x+12}{}$

that make

b) $R(x) = \dfrac{x^2+7x+12}{x+3}$

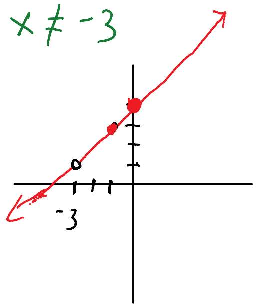
$$R(x) = \frac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}}$$

$$R(x) = (x+4), \quad x \neq -3$$

x	y
-3	1

$(0,4)$

$m = 1$



② Horizontal Asymptotes (H.A.'s)

$$R(x) = \frac{P(x)}{Q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

Assume $R(x)$ and $P(x)$ and $Q(x)$ have no common factors

HA's sometimes occur when $|x|$ becomes large (ie. $x \to \infty$ or $x \to -\infty$)

① If $n < m$, then $\boxed{y = 0}$ is the H.A. (x =axis)

② If $n = m$, then $\boxed{y = \dfrac{a_n}{b_n}}$ is the H.A.

③ If $n > m$, there is no H.A.

(ex) Find the HA's.

a) $R(x) = \dfrac{6x^1 - 2}{2x^2 + 9x - 5}$

leading term test $P(x) = 6x - 2$ behaves same as $6x$ for large $|x|$. (LTT) (so as $x \to \pm\infty$ $P(x) \to (6x)$.

work to show

$n = 1, \quad m = 2$

$n < m$

$n = 1$, $m = 2$

$n < m$

So H.A. is $y = 0$

$P(x) \to 6x$.

For $Q(x) = 2x^2 + 9x - 5$,

$Q(x) \to 2x^2$ for large $|x|$.

So, for large $|x|$, $R(x) \to \frac{6x}{2x^2} = \frac{3}{x} \to 0$

So, the H.A. is $y = 0$

b) $f(x) = \frac{2x^3 - x + 1}{1x^3 - 1}$

$n = 3 = m$

$y = \frac{2}{1}$ H.A.

$y = 2$

③ Slant Asympotes (S.A.)

Also sometimes occur for large $|x|$. Occur when n is exactly 1 unit larger than m.

ex) Find S.A.

$f(x) = \frac{x^2 + x - 6}{x^1 + 2}$

$n = 2$, $m = 1$; so n is 1 larger than m, which means there is a S.A.

which means there is a S.A.

Rule: set $y = \left(\text{Quotient that you get when you long divide } Q(x) \text{ into } P(x)\right)$

All you need to show

$$\begin{array}{r|rrr} -2 & 1 & 1 & -6 \\ & & -2 & 2 \\ \hline & 1 & -1 & -4 \end{array}$$

$y = x - 1$ S.A.

$f(x) = \dfrac{x^2+x-6}{x+2} = x - 1 - \cancel{\dfrac{4}{x+2}} \to 0$ as $|x| \to \infty$

So $f(x)$ gets closer to $y = x - 1$ as $|x| \to \infty$

ex) Graph $\underbrace{f(x)}_{y} = \dfrac{x^2+x-6}{x+2}$

① Intercepts

$\boxed{y\text{-int}}$

$f(0) = \dfrac{-6}{2} = -3$

$(0, -3)$

$\boxed{x\text{-int}}$

$0 = \dfrac{x^2+x-6}{x+2}$

solutions are the same

$0 = x^2+x-6$

$0 = (x-2)(x+3)$

$x = 2, -3$

$(2, 0)$ $(-3, 0)$

② Asymptotes

$\boxed{V.A.}$ $f(x) = \dfrac{(x-2)(x+3)}{x+2}$

$x+2=0$

$\boxed{x=-2}$ V.A.

$\boxed{\text{H.A.}}$ $n=2,\ m=1$

$n>m$

none

$\boxed{\text{S.A.}}$ (see last ex)

$y=x-1$

(3) check for symmetry

~~ex~~.

even fct → y-axis symmetry

odd fctn → origin symmetry (rotate graph about (0,0) 180° it looks the same)

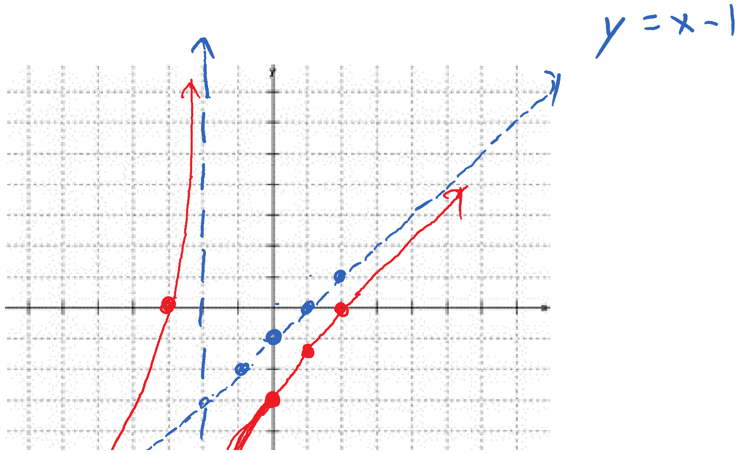
None : $f(-x) \neq f(x)$ not even

$\neq -f(x)$ not odd

(4) Plot points

x	y
-5	$-4.\bar{6}$
-4	-3
-3	0
-1	-6
1	$-\frac{4}{2} \approx -1.6$

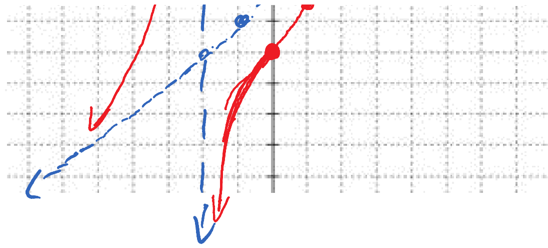
← oh yeah, x-int.



-1	
1	$-\frac{4}{3} \approx -1.6$
2	0
3	1.2

$$\underbrace{f(x)}_{y} = \frac{x^2+x-6}{x+2}$$

$$f(1) = \frac{-4}{3}$$



$x = -2$

(ex) $I(x) = \frac{9}{x+4.5}$ gives current (amps) in a particular circuit, where x is resistance in Ohms. Find and interpret the H.A.

(ex) Sketch the graph of $f(x) = \frac{1}{x^2-9}$

① Intercepts

② Asymptote

③ symmetry

(4) plot some point