Section 4.1: Inverse Functions

Wednesday, February 12, 2014
5:11 PM

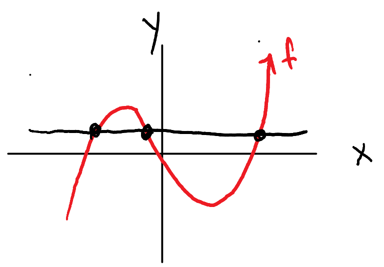
Goals:

1. To find the inverse of a given function
2. To verify using function composition whether or not two functions are inverses.

Recall:

1. A **function** is a set of ordered pairs (inputs and outputs) such that no two different ordered pairs have the same first coordinate. A function passes the **Vertical Line Test**. We usually symbolize y as a function of x by $y = f(x)$.
2. A function is **one-to-one** if no two different ordered pairs have the same second coordinate. A one-to-one function passes the **Horizontal Line Test**.
3. The **inverse** of a function f is obtained by switching the first and second coordinates in all the ordered pairs that comprise f. The inverse of f is denoted by f^{-1}
4. The domain and range of f and f^{-1} are flip-flopped.
5. $f[f^{-1}(x)] = x = f^{-1}[f(x)]$.

ex Is it 1to1?



No, by HLT.

ex Find the inverse

a) $f = \{(1,2)\ (3,4)\ (5,6)\}$

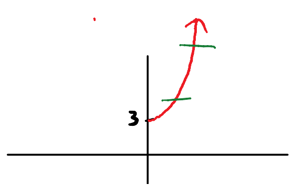
$f^{-1} = \{(2,1)\ (4,3)\ (6,5)$

Domain

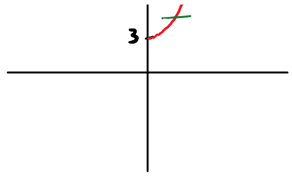
b) $f(x) = x^2+3,\ x \geq 0$

verify f is 1to1

① 1 to 1 by HLT



Note: Domain $[0, \infty)$



Note: Domain $[0, \infty)$

Range $[3, \infty)$

(2) Replace f(x) w/ y

$$y = x^2 + 3$$

(3) switch x and y

$$x = y^2 + 3$$

(4) solve for y

$$y^2 + 3 = x$$

$$\sqrt{y^2} = \pm\sqrt{x - 3}$$

$$y = \pm\sqrt{x-3}$$

take positive root only

$$y = \sqrt{x-3}$$

(5) Replace y w/ $f^{-1}(x)$

$$f^{-1}(x) = \sqrt{x-3}, \quad x \geq 3$$

(Range is $y \geq 0$)

↳ why we didn't take negative root

(ex) let $f(x) = 4 - \frac{1}{x}$ and $g(x) = \frac{1}{4-x}$.

Show f and g are inverses using function composition.

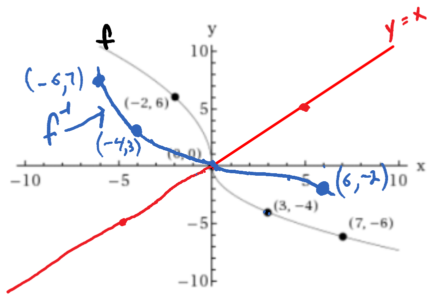
$$(f \circ g)(x) = f[g(x)]$$
$$= f\left[\frac{1}{4-x}\right]$$
$$= 4 - \frac{1}{\left(\frac{1}{4-x}\right)}$$
$$= 4 - 1 \cdot \frac{4-x}{1}$$
$$= 4 - (4-x)$$
$$= x$$

$$(g \circ f)(x) = g[f(x)]$$
$$= g\left[4 - \frac{1}{x}\right]$$
$$= \frac{1}{4-\left(4-\frac{1}{x}\right)}$$
$$= \frac{1}{\cancel{4}-\cancel{4}+\frac{1}{x}}$$
$$= \frac{1}{\frac{1}{x}}$$
$$= x$$

So, f and g are inverses

(ex)

Draw the graph of the inverse relation.

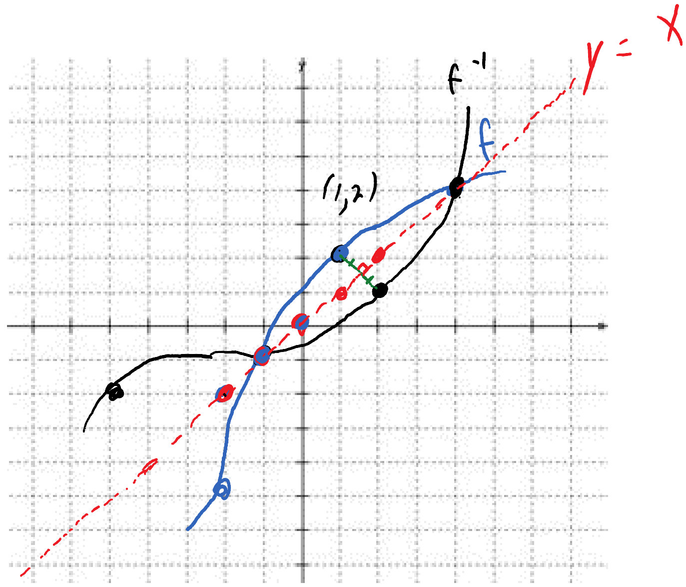


To get f^{-1}, reflect f across $y = x$ (or switch the ordered pairs around in the given points and draw a nice, smooth graph through those new points)

(ex)

x

ex



ex   suppose $f(2) = -5$ and f^{-1} exists.

Name a point on f^{-1}.

$(-5, 2)$