Section 4.3: Logarithmic Functions

Monday, February 24, 2014
5:26 PM

Goal:
1. To evaluate log functions
2. Graph
3. To convert between forms
4. To use in apps

$2^3 = 8$

3 is the ~~exponent~~ on base 2 that produces 8
(logarithm)

$3 = \log_2(8)$

"y equals log base a of x"

Def: $\boxed{a^y = x}$ is equivalent to $\boxed{y = \log_a x}$

exponential form

logarithmic form

Note: ① $f(x) = \log_a x$ is a function

② log fctn (base a) $y = \log_a x$ and exponential fctn (base a) $y = a^x$ are inverses.

inverses

ex convert to its equivalent form

a) $2^4 = 16$ b) $u^v = w$

a) $3^4 = 81$

$4 = \log_3 81$

b) $u^v = w$

$v = \log_u w$

c) $v^{x+y} = u$

$x+y = \log_v(u)$

d) $v = \log_8(w)$

$w = 8^v$

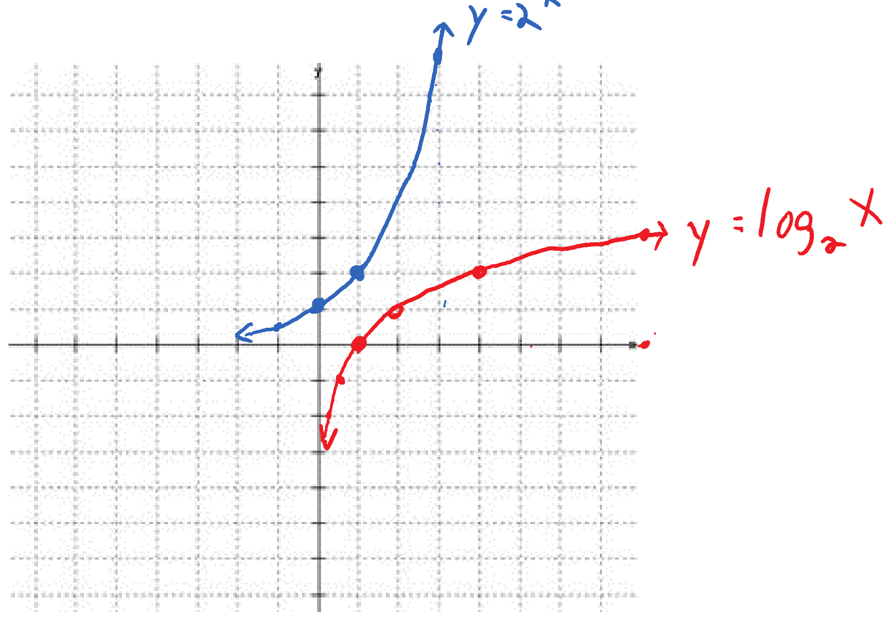
e) $\log_{10}(x+b) = c$

$10^c = x+b$

ex Graph $y = \log_2(x)$ (inverse of $y = 2^x$)

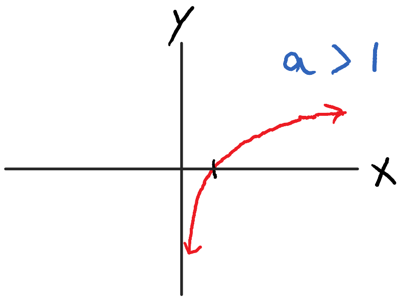
$\boxed{2^y = x}$

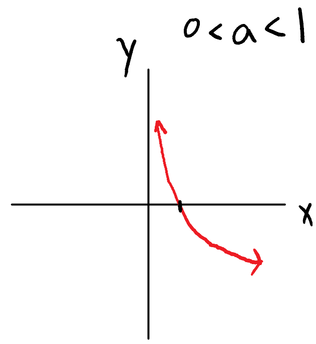
x	y
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3



to graph $\xrightarrow{\text{convert}}$ exp form

$\boxed{y = \log_a x}$ $\boxed{a^y = x}$





Notes on $f(x) = \log_a x$

1. Domain: $(0, \infty)$

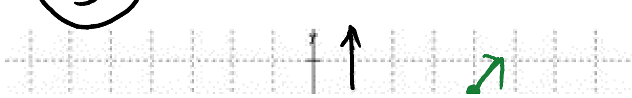
2. Range: $(-\infty, \infty)$

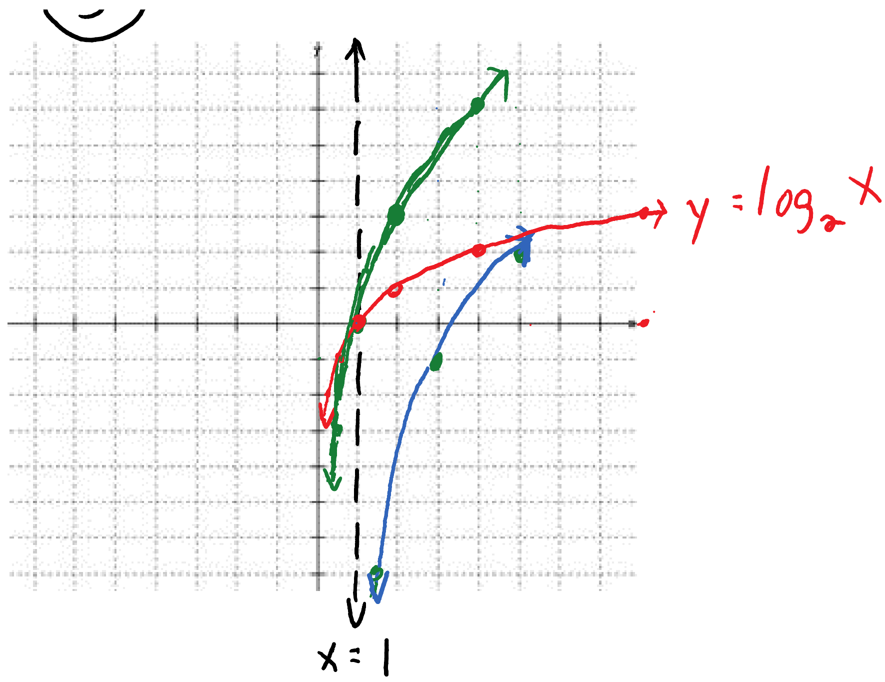
3. $a > 0$, $x > 0$, $a \neq 1$ (restrictions on x, a)

ex) Graph $f(x) = 3\log_2(x-1) - 4$ using transformations.

Base fctn: $y = \log_2(x)$

1. Vertical stretch factor: $|3| = 3$
2. H-shift: $+1$
3. V-shift: -4





Properties of logs and exponents

① $\log_b(b) = 1$ $(b^1 = b)$

② $\log_b 1 = 0$ $(b^0 = 1)$

③ $\log_b(b^x) = x$ $(b^x = b^x)$

☆ ④ $b^x = b^y$ iff $x = y$, $b \neq 0, 1$, $b > 0$

ex solve

a) $\log_4 x = 3$

b) $\log_4 8 = x$

convert to exp form

$x = 4^3$

$x = 64$

$\boxed{4^x = 8}$

$(2^2)^x = 2^3$

$2^{2x} = 2^3$

by #4 $\quad 2x = 3$

$x = \frac{3}{2}$

check : $4^{\frac{3}{2}} \stackrel{?}{=} 8$

$\left(\sqrt[2]{4^3}\right) = \left(\sqrt{4}\right)^3$

$= 2^3$

$= 8$ ✓

ex evaluate without a calculator

$\log_4 64$

let $x = \log_4 64$

$4^x = 64$

$4^x = 4^3$

$x = 3$

Another way

$\log_4 4^3 = 3 \quad$ (by #3)

$$x = 3$$

ex) Find domain

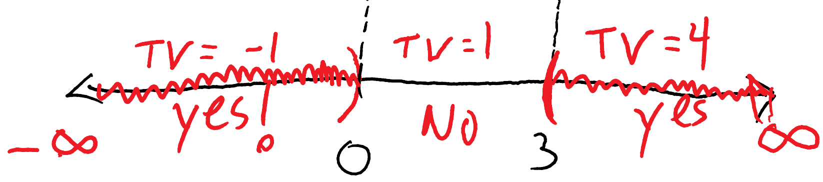
$$f(x) = \log_{10}\left(\frac{x-3}{x}\right)$$

set $\frac{x-3}{x} > 0$

Find critical values (interval endpts) by setting NUM = 0 and DEN = 0 and solving

$x - 3 = 0$, $x = 0$

$x = 3$ $\quad$ $x = 0$ cv's



use test values to determine the intervals of solution.

$$\frac{x-3}{} > 0$$

$\boxed{TV = -1}$ $\qquad$ $\boxed{TV = 1}$ $\qquad$ $\boxed{TV = 4}$

$\frac{(-)}{(-)} \overset{?}{>} 0$ $\qquad$ $\frac{(-)}{+} \overset{?}{>} 0$ NO! $\qquad$ True

$D: (-\infty, 0) \cup (3, \infty)$

Def: ① $\log x$ means $\log_{10} x$ and is called the common logarithm

② $\ln x = \log_e x$ is called the natural logarithm, where $e \approx 2.718$