Section 4.5: Exponential and Logarithmic Equations

Monday, March 03, 2014

Goal: Let's solve these things.

ex solving exponential eqns.

a) $4^{3x} = 32$

$(2^2)^{3x} = 2^5$

$2^{6x} = 2^5$

$6x = 5$

$x = \frac{5}{6}$

b) $\frac{2 \cdot 3^x}{2} = \frac{8}{2}$

$3^x = 4$

$\log(3^x) = \log(4)$

$\frac{x \cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 4}{\log 3}$

$x = \frac{\log 4}{}$

<u>log form</u>

$x = \log_3 4$

$x = \frac{\log 4}{\log 3}$

$$x = \frac{\log 4}{\log 3}$$

c) $9^{7-3x} = 5$

$$\log 9^{7-3x} = \log 5$$

$$(7-3x)\log 9 = \log 5$$

$$\cancel{7\log 9} - 3x\log 9 = \log 5$$
$$\cancel{-7\log 9} \qquad -7\log 9$$

$$\frac{-3x\cancel{\log 9}}{\cancel{-3\log 9}} = \frac{\log 5 - 7\log 9}{-3\log 9}$$

$$x = \frac{\log 5 - 7\log 9}{-3\log 9}$$

$$x = \frac{7\log 9 - \log 5}{3\log 9}$$

☆ d) $5^x = 3^{x+1}$

$$\log 5^x = \log 3^{x+1}$$

$$x \log 5 = (x+1)\log 3$$

$$x \log 5 = x \cancel{\log 3} + \log 3$$

$$-x\log 3 \quad -x\cancel{\log 3}$$

$$x\log 5 - x\log 3 = \log 3$$

$$\frac{x\cancel{(\log 5 - \log 3)}}{\cancel{(\log 5 - \log 3)}} = \frac{\log 3}{\log 5 - \log 3}$$

$$x = \frac{\log 3}{\log 5 - \log 3}$$ exact

$x \doteq 2.15$ estimate

Steps

1. isolate exp. fctn if necessary
2. If possible, write both sides w/ same base, set exponents equal and solve
3. If 2 fails, take log (or ln) of both sides, apply power rule, and solve.

ex non-routine equation

$$\frac{e^x - e^{-x}}{2} = 15$$

$$e^x - e^{-x} = 30$$

$$e^x\left(e^x - \frac{1}{e^x}\right) = e^x \cdot 30$$

$$(e^x)^2 - 1 = 30e^x$$

$$1(e^x)^2 - 30e^x - 1 = 0$$ Think quadratic equation w/ unknown e^x

$a = 1$, $b = -30$, $c = -1$

x $\quad 30 \pm \sqrt{900 - 4(1)(-1)}$

$$e^x = \frac{30 \pm \sqrt{900 - 4(1)(-1)}}{2}$$

$$e^x = \frac{30 \pm \sqrt{904}}{2}$$

can't be negative
(so throw out minus)

$$\ln e^x = \ln\left(\frac{30 + \sqrt{904}}{2}\right)$$

$$x = \ln\left(\frac{30 + \sqrt{904}}{2}\right)$$

$$= \ln\left(\frac{30 + 2\sqrt{226}}{2}\right)$$

$$x = \ln\left(15 + \sqrt{226}\right)$$

ex Logarithmic Equations

(1) 1 to 1 property on logarithms

$$\log_a x = \log_a y \quad \text{iff} \quad x = y$$

* ② Def of log

$$x = a^y \quad \text{iff} \quad y = \log_a x$$

ex solve

a) $\log_3 x = 4$

$x = 3^4$

$x = 81$

b) $\log_4(x-3) = \log_4 5$

$x - 3 = 5$

$x = 8$

c) $\dfrac{4 \log x}{4} = \dfrac{8}{4}$

$\log_{10} x = 2$

$x = 10^2$

$x = 100$

d) $\log(x-9) + \log x = 1$

$\log_{10}[(x-9)x] = 1$

$(x-9)x = 10^1$

$x^2 - 9x = 10$

$x^2 - 9x - 10 = 0$

$$(x-10)(x+1)=0$$

$$x = 10 \quad \text{or} \quad \cancel{x = -1}$$

e) $\log_6(x+3) - \log_6(x+2) = \log_6 20$

$$\log_6\left(\frac{x+3}{x+2}\right) = \log_6 20$$

$$\frac{x+3}{x+2} = \frac{20}{1}$$

$$20x + 40 = x + 3$$

$$19x = -37$$

$$x = \frac{-37}{19}$$

<u>Solving log equations</u>

case ① Get a single log on one side then convert to exp. form and solve

or get a single log on both sides

case 2 Get ... logs equa
then set the arguments of logs equal
(drop the logarithms) and solve