Section 5.4 Part 2: Trig Functions of Real Numbers

Monday, August 25, 2014
5:20 PM

Goals: To evaluate a trig function of any real number.

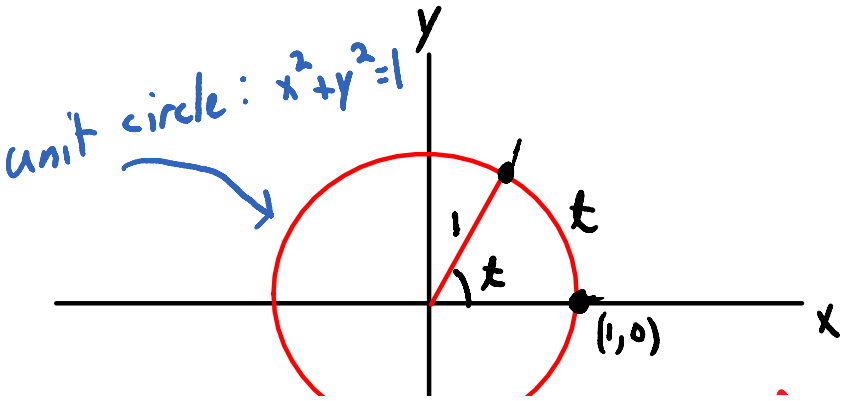
Notes: Applications of periodic functions include…
1. Spring vibrations
2. Tides (water depth at a location)
3. Outside temperature throughout the day
4. AC current

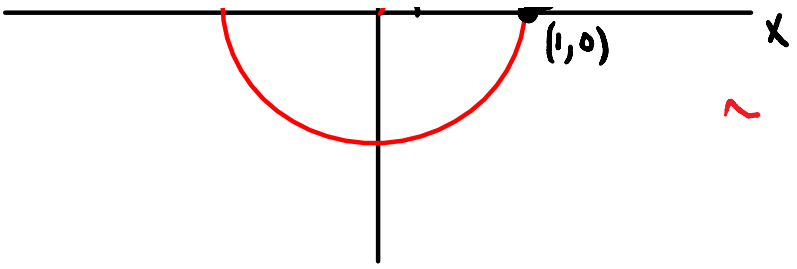
Big Idea: We can use trigonometric functions of real numbers to model repetitive phenomena.

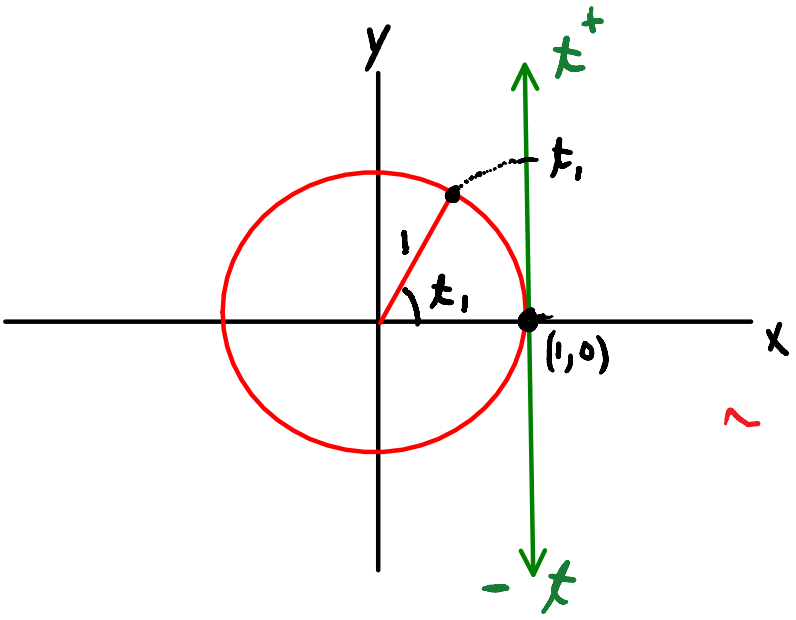
The Wrapping Function

Recall radian measure: $\theta = \frac{s}{r}$

So, on the unit circle: $\theta = s$







wrapping fctn: $W(t) = (x,y)$

a point on the unit circle

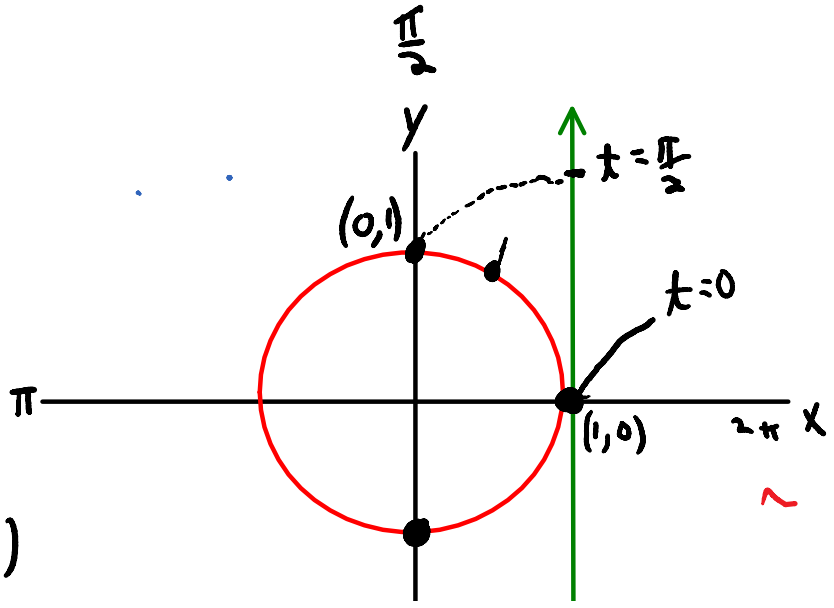
(ex) Find t

$W(0) = (1,0)$

$W\left(\frac{\pi}{2}\right) = (0,1)$

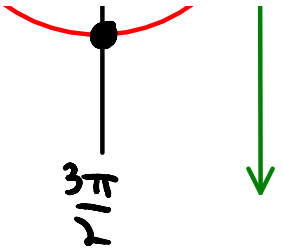
$W(\pi) = (-1,0)$

$W\left(\frac{3\pi}{2}\right) = (0,-1)$



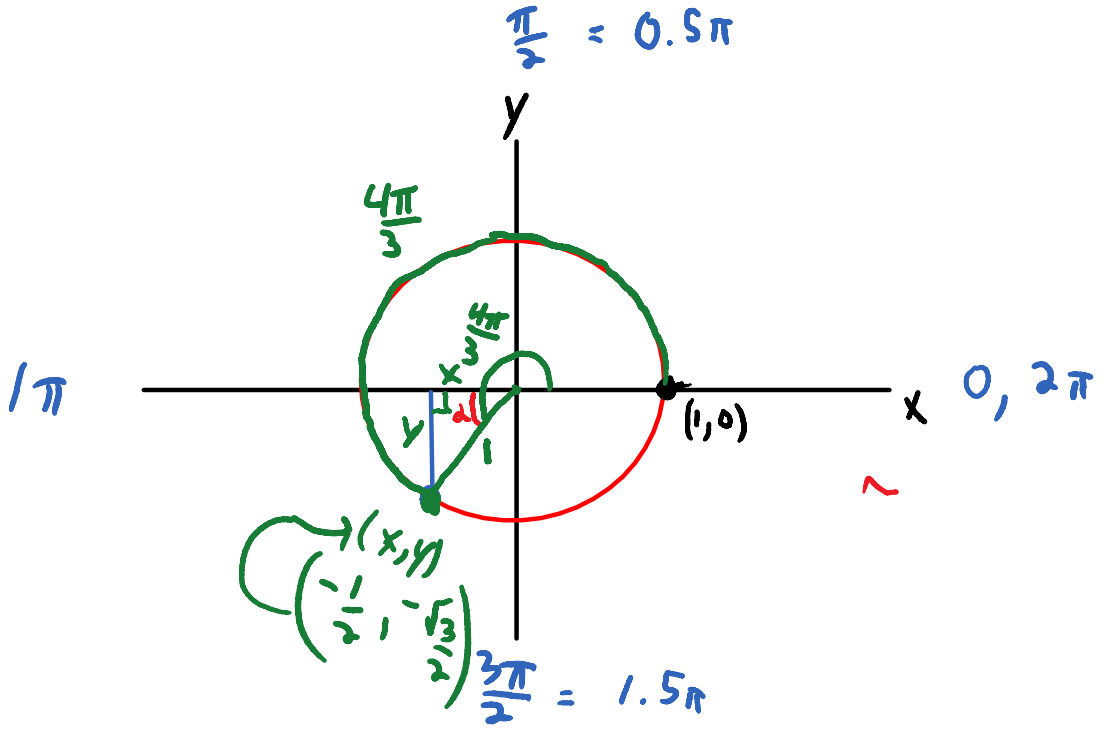
$w\left(\frac{3\pi}{2}\right) = (0,-1)$

$w(2\pi) = (1,0)$



(ex) Find $w(t)$ for

$t = \frac{4\pi}{3}$



$x = \cos\frac{4\pi}{3}$

$y = \sin\frac{4\pi}{3}$

scratch

$\alpha = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

$x = \cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$$y = \sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$W\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

Def: Trig fctns of a real # t

Let $W(t) = (x,y)$, where (x,y) on unit circle. Then....

$\sin t = y$

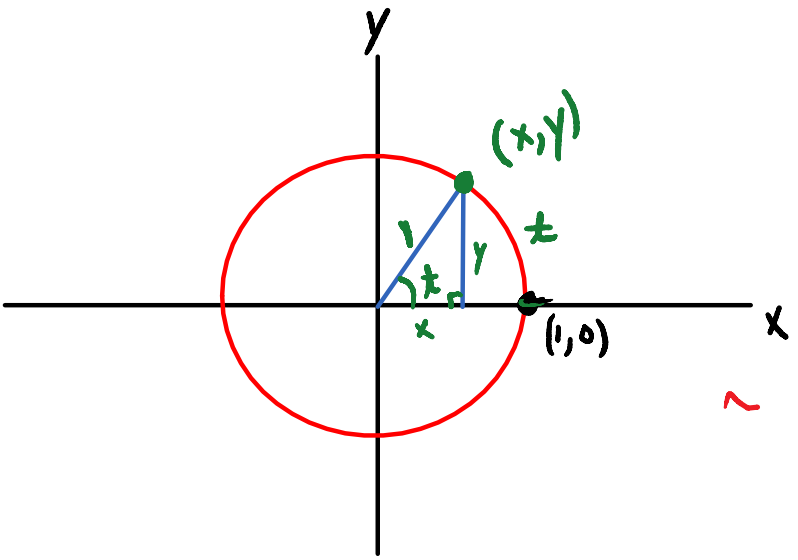
$\cos t = x$

$\tan t = y/x$

$\cot t = x/y$

$\sec t = 1/x$

$\csc t = 1/y$



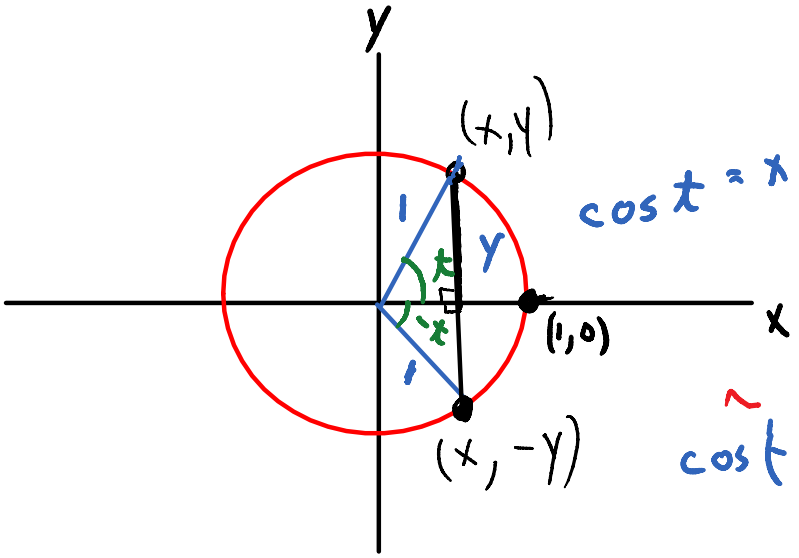
Recall: ① even fctn, f: $f(-x) = f(x)$

② odd fctn, f: $f(-x) = -f(x)$

Note: cosine and secant are even, the rest are odd.

(ex) even: $\cos(-t) = \cos t$

odd: $\sin(-t) = -\sin t$



$\cos t = x = \cos(-t)$

So, cosine is an even function.

Note: ① A function is periodic if there is a smallest number p such that $f(t+p) = f(t)$

period.

② The period of sine, cosine, secant and cosecant is 2π

(3) The period of tangent and cotangent.

(ex) Is it even, odd, or neither?

a) $f(x) = \dfrac{\cos x}{x}$

$$f(-x) = \frac{\cos(-x)}{-x}$$

$$= \frac{\cos x}{-x}$$

$$= -\frac{\cos x}{x}$$

$$= -f(x)$$

f odd

e. Fundamental Identities

$\sin t = \dfrac{1}{\csc t}$, $\cos t = \dfrac{1}{\sec t}$, $\tan t = \dfrac{1}{\cot t}$

$$\tan t = \frac{\sin t}{\cos t}$$

$\tan t = \frac{\sin t}{\cos t}$

$\cos^2 t + \sin^2 t = 1$ → $\cos^2 t = 1 - \sin^2 t$

(ex) Write as a single trig fctn.

$$\frac{1 - \sin^2 t}{\cot^2 t}$$

$$= \frac{\cos^2 t}{\frac{\cos^2 t}{\sin^2 t}}$$

$$= \cos^2 t \cdot \frac{\sin^2 t}{\cos^2 t}$$

$$= \sin^2 t$$