Section 6.1: Trigonometric Identities

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Goal: To verify trig identities

Homework is out of the text (or etext on webassign) at the end of section 6.1
p. 524 1-69 e.o.o. (1, 5, 9, ... 69)

A trig identity is an equation that is always true for all relevant x-values

$\sin x = \tan x \cos x$ identity

$$\tan x \cos x = \frac{\sin x}{\cos x} \cdot \cos x$$

$$= \sin x$$

Done

Basic Identities

$\sin x = \frac{1}{\csc x}$ $\cos x = \frac{1}{\sec x}$ $\tan x = \frac{1}{\cot x}$

$\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

* $\cos^2 x + \sin^2 x = 1$
$1 - \cos^2 x = \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$, $\cot^2 x + 1 = \csc^2 x$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \rightarrow 1 + \tan^2 x = \sec^2 x$$

(ex) Verify

a) $\sin x \cot x \sec x = 1$

$$\sin x \cot x \sec x = \sin x \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$$

$$= 1$$

Done

b) $\frac{1}{\sin x} + \frac{3}{\cos x} = \frac{\cos x + 3\sin x}{\sin x \cos x}$

$$\frac{1 \cdot \cos x}{\sin x \cdot \cos x} + \frac{3 \cdot \sin x}{\cos x \cdot \sin x} = \frac{\cos x}{\sin x \cos x} + \frac{3\sin x}{\sin x \cos x}$$

$$= \frac{\cos x + 3\sin x}{\sin x \cos x}$$

Done

c) $\frac{\sin^2 x - 2\sin x + 1}{\sin x - 1} = \sin x - 1$

$$\frac{\sin^2 x - 2\sin x + 1}{\sin x - 1} = \frac{(\sin x - 1)(\sin x - 1)}{(\sin x - 1)}$$

$$= \sin x - 1$$

Done

d) $\sec^2 x + 2\tan x = (\tan x + 1)^2$

$(A \pm B)^2 = A^2 \pm 2AB + B^2$

$(\tan x + 1)^2 = \tan^2 x + 2\tan x + 1$

$= (\tan^2 x + 1) + 2\tan x$

$= \sec^2 x + 2\tan x$

Done

e) $\dfrac{\sin t}{1+\cos x} = \csc x - \cot x$

$\dfrac{\sin x\,(1-\cos x)}{(1+\cos x)(1-\cos x)} = \dfrac{\sin x(1-\cos x)}{1-\cos^2 x}$

$A + B$, $A - B$ conjugates

$(A+B)(A-B) = A^2 - B^2$

$= \dfrac{\sin x \cdot (1-\cos x)}{\sin^2 x}$

$= \dfrac{1-\cos x}{\sin x}$

$= \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}$

$= \csc x - \cot x$

Done!

f) $\dfrac{2\sin x \cot x + \sin x - 4\cot x - 2}{2\cot x + 1} = \sin x - 2$

$\dfrac{2\sin x \cot x + \sin x - 4\cot x - 2}{2\cot x + 1} = \dfrac{\sin x(2\cot x + 1) - 2(2\cot x + 1)}{2\cot x + 1}$

$$= \frac{(\sin x - 2)(3\cot x + 1)}{(3\cot x + 1)}$$

$$= \sin x - 2$$

Done!

Helpful

∧ Factoring Formulas

$$A^2 - B^2 = (A+B)(A-B)$$

$$A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$$

ex Prove

$$\frac{1+\tan^3 x}{1+\tan x} = 1 - \tan x + \tan^2 x$$

$$\frac{1+\tan^3 x}{1+\tan x} = \frac{(1+\tan x)(1-\tan x+\tan^2 x)}{1+\tan x}$$

$$= 1 - \tan x + \tan^2 x$$

Done

Identity Verification Guidelines

1. Work with the side with more "stuff."
2. Perform operations (+, -, x, squaring) or factor.
3. Use established identities.
4. Change to sines and cosines.

5. Mulitply by a special form of 1 (e.g. multiply a numerator and denominator by a conjugate).
6. Look at the other side of the equal sign to see if you are headed in the right direction.

Note: These guidelines can be helpful but they are not written in stone, so be flexible. Sometimes, for example, you will work with the side with less "stuff."