Section 6.3: Double and Half-Angle Identities

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Goal:	To verify identities involving the Double and Half-Angle formulas.

Assignment: 1-89 e.o.o, 91

Double Angle Identities

① $\sin(2\theta) = 2\sin\theta\cos\theta$

② $\cos(2\theta) = \overbrace{\cos^2\theta}^{1-\sin^2\theta} - \overbrace{\sin^2\theta}^{(1-\cos^2\theta)}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

③ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Proof: ① $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin(2\theta) = \sin(\overset{\alpha}{\theta} + \overset{\beta}{\theta})$

$= 1\sin\theta\cos\theta + 1\sin\theta\cos\theta$

$= 2\sin\theta\cos\theta$

Done.

ex Write in terms of a single trig fctn:

$2\cos^2(\underbrace{2\beta}_{\theta}) - 1$ | $\cos 2\theta = 2\cos^2\theta - 1$

$\cos[2(2\beta)]$

$\cos(4\beta)$

Power Reducing Identities

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta} = \frac{(1-\cos 2\theta)^2}{\sin^2 2\theta} = \frac{\sin^2 2\theta}{(1+\cos 2\theta)^2}$$

Half-Angle Identities. Let $\theta = \frac{\alpha}{2}$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

ex Find the exact value

a) $\cos 105^\circ$

$$= \cos\left(\frac{210^\circ}{2}\right)$$

$$= -\sqrt{\frac{1+\cos 210^\circ}{2}}$$

$$= -\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= -\sqrt{\frac{\left(1-\frac{\sqrt{3}}{2}\right)}{2}\cdot\frac{2}{2}}$$

$$= -\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2-\sqrt{3}}}{2}$$

b) $\sin\left(\frac{3\pi}{8}\right)$

$$2\cdot\frac{3\pi}{8} = \frac{3\pi}{4}$$

$$= \sin\left(\frac{\frac{3\pi}{4}}{2}\right)$$

$$= \sqrt{\frac{1-\cos\frac{3\pi}{4}}{2}}$$

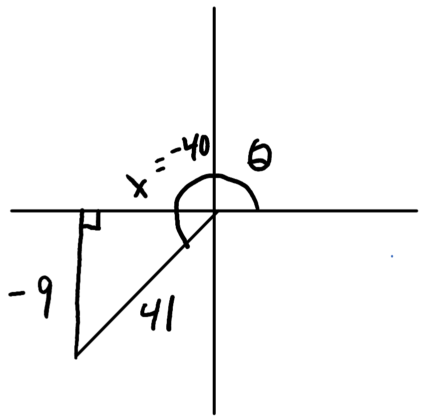
$$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\left(1+\frac{\sqrt{2}}{2}\right)}{2}\cdot\frac{2}{2}}$$

$$= \frac{\sqrt{2+\sqrt{2}}}{2}$$

ex Given: $\sin\theta = -\frac{9}{41}$ θ in Q3

Find $\sin 2\theta$.



$$x^2 + (-9)^2 = 41^2$$

$$x^2 + 81 = 41^2$$

$$\sqrt{x^2} = \sqrt{1600}$$

$$x = -40$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$= 2\left(\frac{-9}{41}\right)\left(\frac{-40}{41}\right)$$

$$= \frac{720}{1681}$$

ex Verify

a) $\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \frac{1}{2} \csc x \sin 2x$

$$\frac{1}{2} \cdot \frac{1}{\sin x} \cdot 2 \sin x \cos x$$

$$\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = \cos\left(2 \cdot \frac{x}{2}\right)$$

$$= \cos x$$

$$= \frac{1}{2} \cdot 2 \frac{\sin x}{\sin x} \cdot \cos x$$

$$= \frac{1}{2} \cdot \frac{1}{\sin x}\left(2 \sin x \cos x\right)$$

$$= \frac{1}{2} \csc x \sin 2x$$

Done