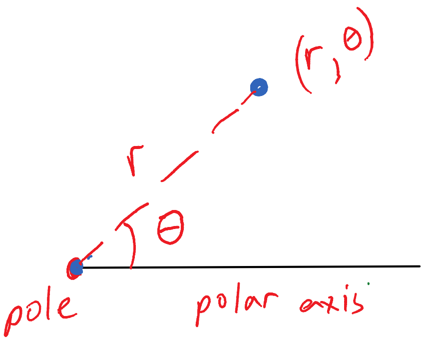
Section 8.5: Polar Coordinates

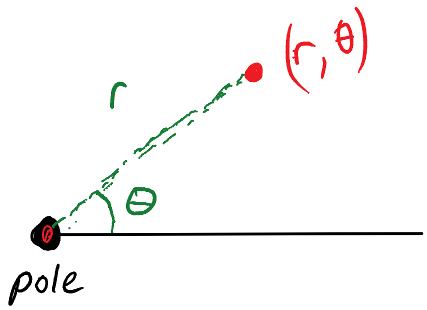
Friday, March 28, 2014
2:12 PM

Goals:

1. To convert between polar and rectangular coordinates.
2. To graph in polar coordinates.

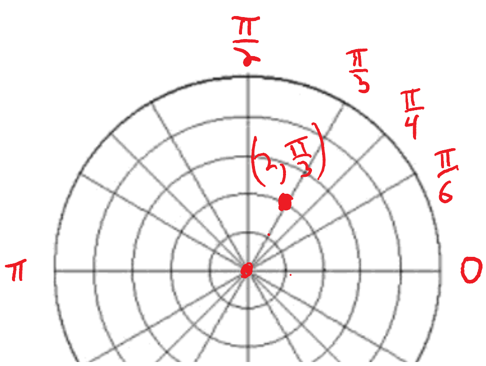
Another system for locating points using a radius and an angle (r, θ)

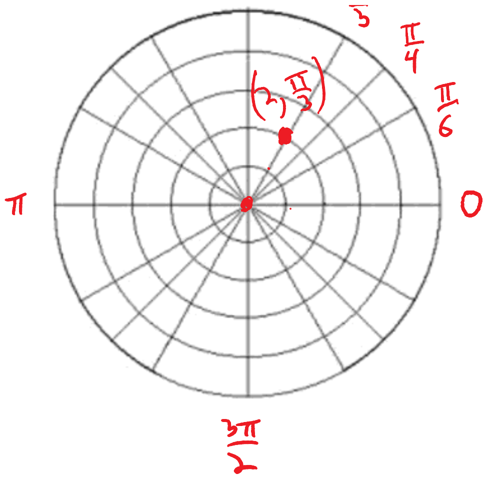




ex plot the points

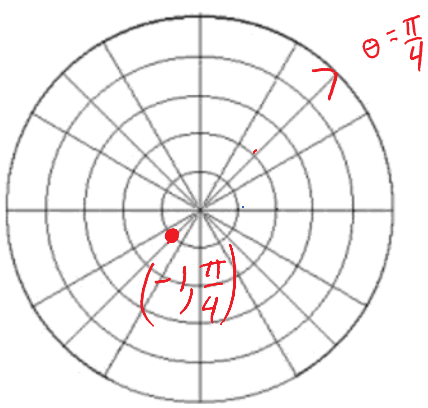
a) $(\overset{r}{2}, \overset{\theta}{\frac{\pi}{3}})$





b) $\overset{r}{(-1},\ \overset{\theta}{\frac{\pi}{4})}$

Not unique



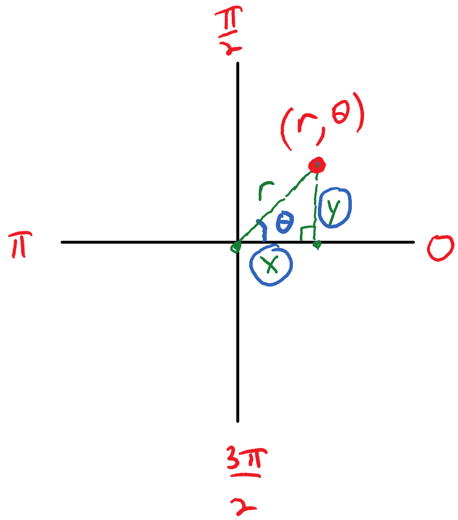
coordinate conversions

$$r^2 = x^2 + y^2$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$



ex convert to rectangular

a) $(r, \theta) = \left(\overset{r}{5}, \overset{\theta}{\frac{\pi}{6}}\right)$

$$x = r\cos\theta = 5\cos\left(\frac{\pi}{6}\right) = 5 \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$$

$$y = r\sin\theta = 5\sin\frac{\pi}{6} = 5 \cdot \frac{1}{2} = \frac{5}{2}$$

$$\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$$

b) $r = \cos\theta$

... lt both sides by r (assume $r \neq 0$)

$\cdot r$ mult. both sides by r (assume $r \neq 0$)

$r^2 = r\cos\theta$

$x^2 + y^2 = x$

$-x \qquad -x$

$(x^2 - 1x + \frac{1}{4}) + y^2 = 0 + \frac{1}{4}$

$(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$

circle std form

$\boxed{(x-h)^2 + (y-k)^2 = r^2}$

$c\ (\frac{1}{2}, 0)$

radius $= \frac{1}{2}$

c) $r = \cos\theta + 2\sin\theta$

$\cdot r$

$\underbrace{r^2}_{x^2+y^2} = \underbrace{r\cos\theta}_{x} + 2\underbrace{r\sin\theta}_{y}$

$x^2 + y^2 = x + 2y$

convert to polar

(ex) Conver...

a) $(x,y) = (\overset{x}{-1}, \overset{y}{1})$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-1)^2 + 1^2}$

$r = \sqrt{2}$

$\tan\theta = \frac{1}{-1} = -1$

$\tan\theta = -1$

$\alpha = -45^0$

$\theta = 180^0 - 45^0$

$\theta = 135^0$

$(\sqrt{2}, 135^0)$

b) $x = -4$

$\frac{r\cancel{\cos}\theta}{\cancel{\cos}\theta} = \frac{-4}{\cos\theta}$

$r = -4\sec\theta$

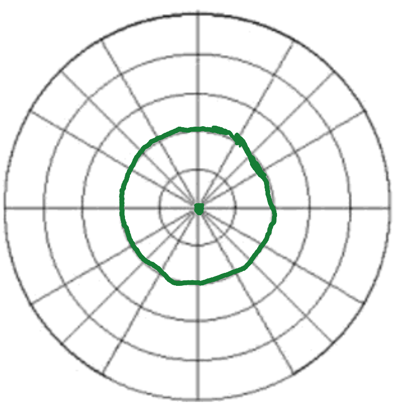
c) $y^2 = 4y$

$(r\sin\theta)^2 = 4r\sin\theta$

$\boxed{r^2\sin^2\theta = 4r\sin\theta}$

ex Graph

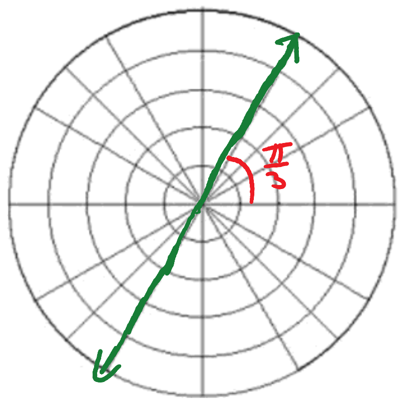
a) $r = 2$



circle C$(0,0)$
radius $= 2$

b) $\theta = \frac{\pi}{3}$

r can be any



r can be any ~~Length~~ value

Special Polar Graphs

→ $\boxed{\text{Limacon}}$

sym. to polar axis

sym. to $\theta = \frac{\pi}{2}$

$r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$, a and b constants

$0 < \left\lvert\frac{a}{b}\right\rvert < 1$	$\left\lvert\frac{a}{b}\right\rvert = 1$	$1 < \left\lvert\frac{a}{b}\right\rvert < 2$	$\left\lvert\frac{a}{b}\right\rvert > 2$
inner loop	cardioid	dimple	

$r = a + b\sin\theta$

Rose Curves

sym. to $\theta = \frac{\pi}{2}$ (y-axis)

sym. to polar axis (x-axis)

$r = a\sin(n\theta)$ or $r = a\cos(n\theta)$, where

n is a counting number > 1. $(n = 2, 3, 4, \ldots)$

If n is odd $\rightarrow$ n petals

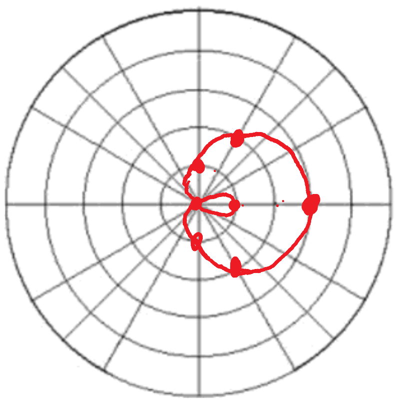
n is even $\rightarrow$ $2n$ petals

ex Graph

a) $r = 1 + 2\cos\theta$ $\quad \pi/3$

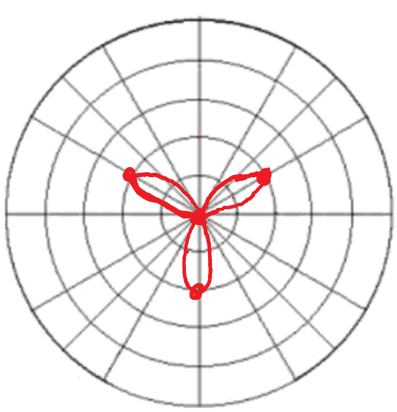
sym to the polar axis

$\left|\frac{a}{b}\right| = \frac{1}{2}$



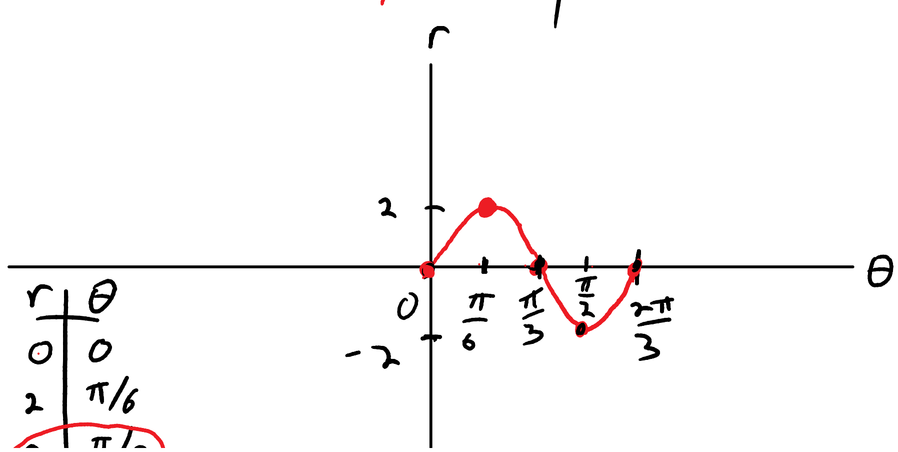
r	θ
3	0
2	$\pi/3$
1	$\pi/2$
0	$\frac{2\pi}{3}$
-1	π
1	$3\pi/2$
3	2π

b) $r = 2\sin 3\theta$

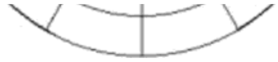


treat $\overset{y}{r} = 2\sin 3\overset{x}{\theta}$ like a rectangular equation

$P = \frac{2\pi}{3}$



r	θ
0	0
2	$\pi/6$



r	θ
0	0
2	π/6
0	π/3
−2	π/2
0	2π/3

−2

c) $r = 3\cos 2\theta$

0

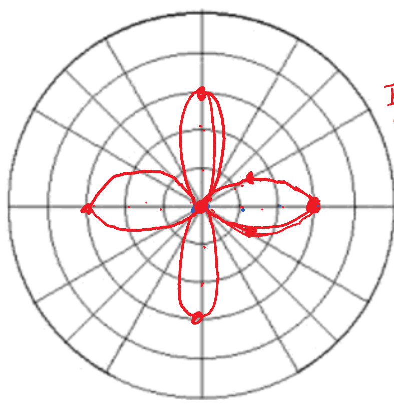
$\frac{\pi}{4}$ $\frac{\pi}{6}$

$2\theta = \frac{\pi}{3}$

$n = 2$

4 petals

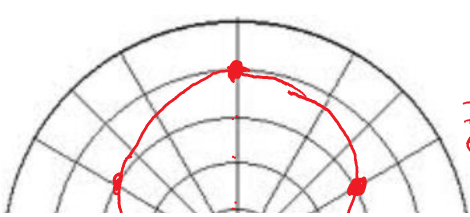
$\frac{360^0}{4} = 90^0$



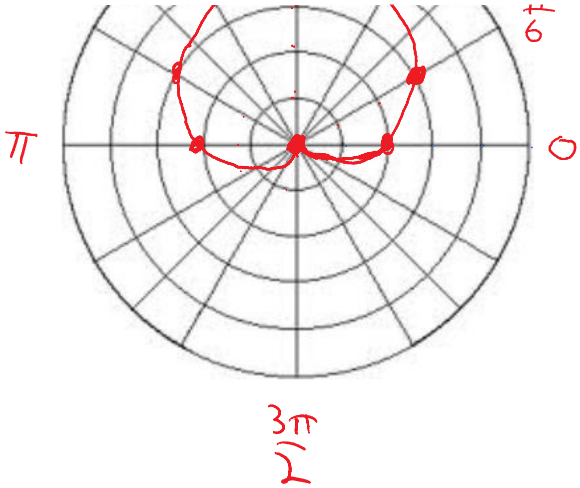
r	θ
3	0
$\frac{3}{2}$	π/6
0	$\frac{\pi}{4}$

d) $r = 2 + 2\sin\theta$

$\pi/6$



r	θ
2	0
3	π/6



3	$\pi/6$
4	$\frac{\pi}{2}$
2	π
0	$\frac{3\pi}{2}$
2	2π

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\boxed{A = Pe^{rt}}$$ continuous compounding

$$\cancel{A} =$$

$$\boxed{N(t) = N_0 e^{kt}}$$