Section 9.1, 9.3: Solving Systems of Two Equations, Two Unknowns

Wednesday, March 26, 2014
6:08 PM

Goals:

1. To solve a linear system of two equations, two unknowns (6.1)
2. To solve a non-linear system of two equations, two unknowns (6.3).

9.1 ex) solve using substitution

a) $\begin{cases} -3x+7y=14 \\ 2x-y=-13 \end{cases}$

$y = 2x+13$

$-3x+7(2x+13)=14$

$-3x+14x+91=14$

$11x=-77$

$x=-7$

$y=2(-7)+13=-1$

$(-7,-1)$ intersect pt.

b) Same system, use elimination to solve

$-3x+7y=14$

$7(2x-y=-13)$

$$\begin{array}{r} 14x-7y=-91 \\ -3x+7y=14 \\ \hline 11x=-77 \end{array}$$

$x=-7$

$-14-y=-13$

$y = -1$

$(-7, -1)$

c) solve

$$\left.\begin{aligned} y &= 2x - 7 \\ 4x - 2y &= 14 \end{aligned}\right\}$$

lines coincide (every point on the line is a solution to system)

$4x - 2(2x - 7) = 14$

$\cancel{4x} - \cancel{4x} + 14 = 14$

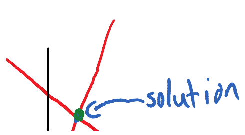
$14 = 14$ True → Dependent system

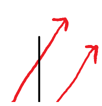
How to write the solutions for Dependent System

set builder notation $\{(x, y) \mid y = 2x - 7\}$

ordered pair $(x, 2x - 7) \rightarrow (c, 2c - 7)$

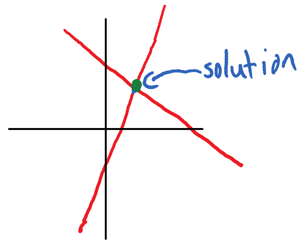
3 situations

1 

2 

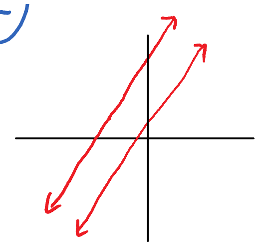
False statement

1



system is consistent and independent

2



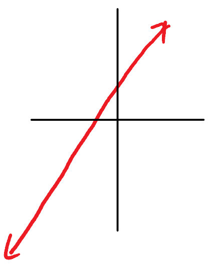
False statement

→ no solution

- parallel lines
- no solution

inconsistent system

3



Lines overlap.

Every ordered pair is a solution.

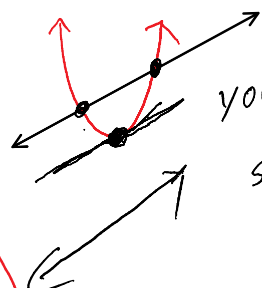
system is consistent ↳ at least 1 solution

and dependent

9.3 Equations are not necessarily linear. (solutions are still intersection points).

ex solve

a) $\begin{cases} y = x^2 + 2x - 3 \\ y = x - 1 \end{cases}$



you can 0, 1, or 2 solutions

use substitution

$x - 1 = x^2 + 2x - 3$

$0 = 1x^2 + 1x - 2$

$0 = (x+2)(x-1)$

$x = -2$ or $x = 1$

$\cancel{y} = 0$

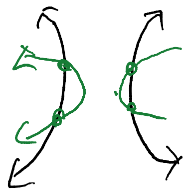
$y = -2 - 1$

$y = -3$

$(-2, -3) \qquad (1, 0)$

b) $3x^2 - 2y^2 = 19$ ← hyperbola

$-2\left(x^2 - y^2 = 5\right)$ ← hyperbola



As many as 4 solations

$$\begin{aligned} -2x^2 + \cancel{2y^2} &= -10 \\ 3x^2 - \cancel{2y^2} &= 19 \\ \hline x^2 &= 9 \end{aligned}$$

$x = \pm 3$

$(\pm 3)^2 - y^2 = 5$

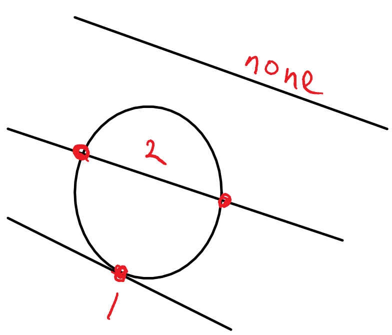
$9 - y^2 = 5$

$y^2 = 4$

$y = \pm 2$

$\cancel{(\pm 3, \pm 2)}$ $\quad (\pm 3, -2), (\pm 3, 2)$

c) $\begin{cases} (x+2)^2 + (y-2)^2 = 13 \\ 2x + y = 6 \end{cases}$



$y = 6 - 2x$

$(x+2)^2 + ((6-2x) - 2)^2 = 13$

$$(x+2)^2 + ((6-2x)-2)^2 = 13$$

$$(x+2)^2 + (4-2x)^2 = 13$$

$$\cancel{x^2} + \cancel{4x} + 4 + 16 - \cancel{16}x + \cancel{4x^2} = 13$$

$$5x^2 - 12x + 20 = 13$$

$$5x^2 - 12x + 7 = 0$$

$$(5x-7)(x-1) = 0$$

$$5x-7=0 \quad \text{or} \quad x-1=0$$

$$x = \frac{7}{5} \quad \text{or} \quad x = 1$$

$$y = 6 - 2\left(\frac{7}{5}\right) \qquad y = 6-2$$

$$y = 6 - \frac{14}{5} \qquad y = 4$$

$$= \frac{30}{5} - \frac{14}{5}$$

$$= \frac{16}{5}$$

$$\left(\frac{7}{5}, \frac{16}{5}\right) \quad \text{or} \quad (1, 4)$$

absolute value

$$\{ a, \quad a \geq 0$$

← called a piece-wise defined function

$$\boxed{|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}}$$

Def

defined function

(ex) Represent the distance between x and 10 using abs. value.

$|x-10|$ (same as $|10-x|$)