Trig Functions of Real Numbers

Goals: To evaluate a trig function of any real number.

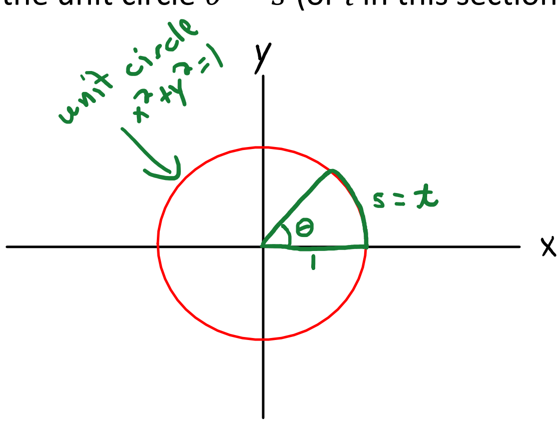
Notes: Applications of periodic functions include…

1. Spring vibrations
2. Tides (water depth at a location)
3. Outside temperature throughout the day
4. AC current

Big Idea: We can use trigonometric functions of real numbers to model repetitive phenomena.

Notes:

1. Recall radian angle measure: $\theta = s/r$.
2. So, on the unit circle $\theta = s$ (or t in this section).

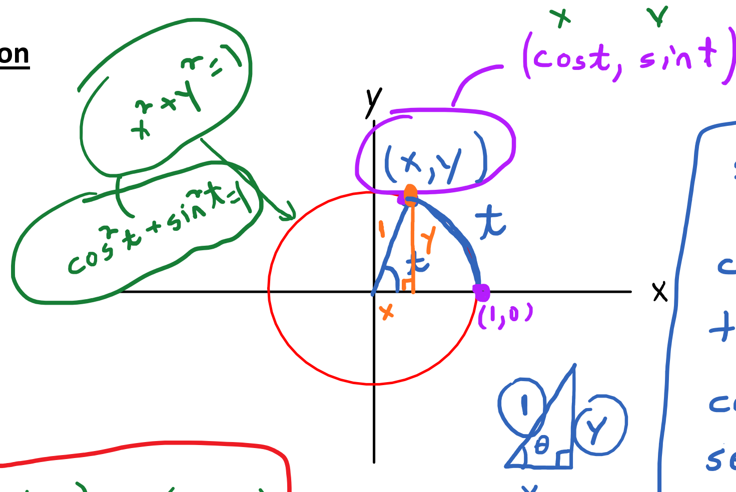


$\theta = \frac{s}{r} = \frac{s}{1} = s$

$\boxed{\theta = t}$

on unit circle

The Wrapping Function



t = real #

$\sin t = y$

$\cos t = x$

$\tan t = y/x$

$\cot t = x/y$

$\sec t = 1/x$

$W(t) = (x, y)$

$\sec t = 1/x$

$\csc t = 1/y$

Wrapping Function: $\boxed{w(t) = (x, y)}$

a point on the unit circle

Example: Find…

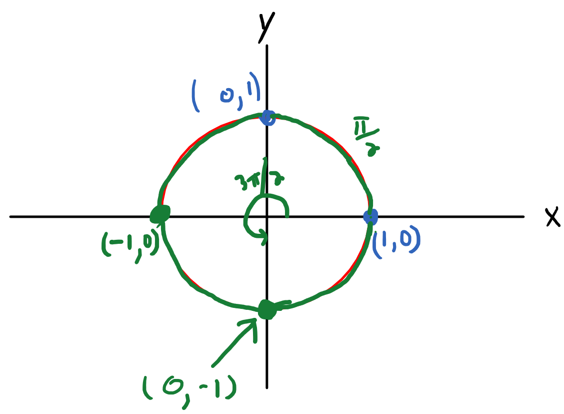
$w(0) = (1, 0)$

$w\left(\frac{\pi}{2}\right) = (0, 1)$

$w(\pi) = (-1, 0)$

$w\left(\frac{3\pi}{2}\right) = (0, -1)$

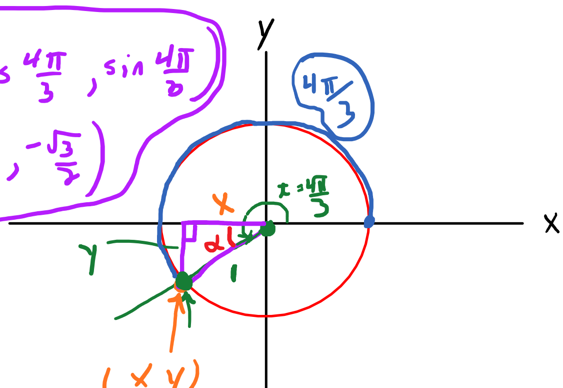
$w(2\pi) = (1, 0)$



Example: Find $w(4\pi/3)$.

$w\left(\frac{4\pi}{3}\right) = \left(\cos\frac{4\pi}{3}, \sin\frac{4\pi}{3}\right)$

$= \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



$\cos\frac{4\pi}{3}$ $\sin\frac{4\pi}{3}$

$w\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\alpha = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

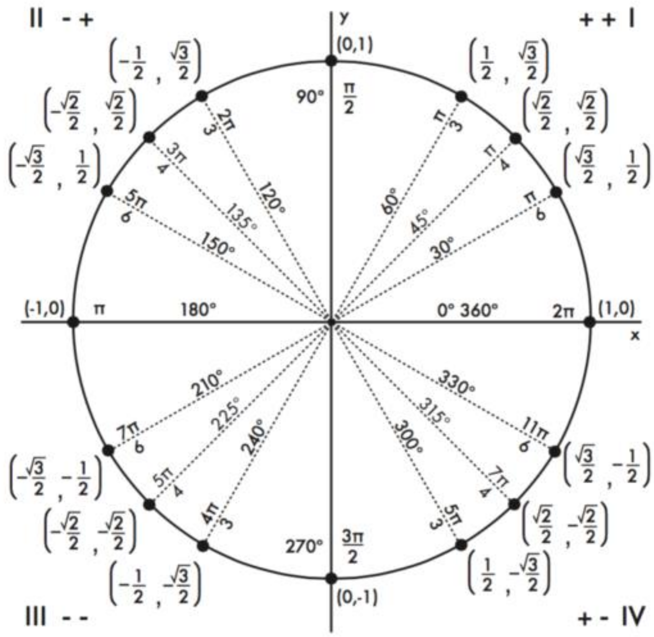
$\cos\frac{\pi}{3} = \frac{1}{2}$

$x = \cos\frac{4\pi}{3} = -\frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$y = \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

Note: $w(t) = \underline{(\overset{x}{\cos t}, \overset{y}{\sin t})}$



Definition: The **trigonometric functions** of a real number, t. Let $w(t) = (x, y)$, where (x, y) is on the unit circle. Then…

$\sin t = y$

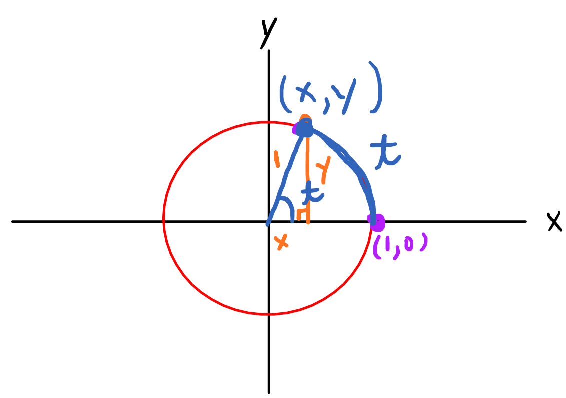
$\cos t = x$

$\tan t = y/x$

$\cot t = x/y$

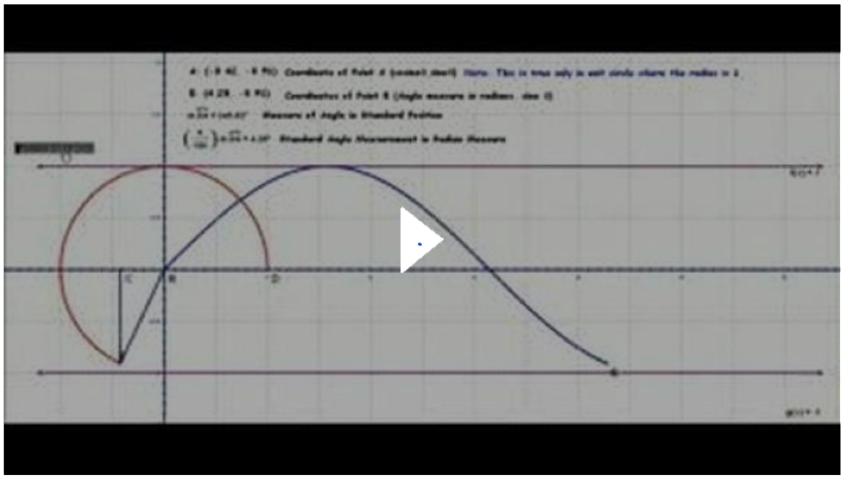
$\sec t = 1/x$

$\csc t = 1/y$

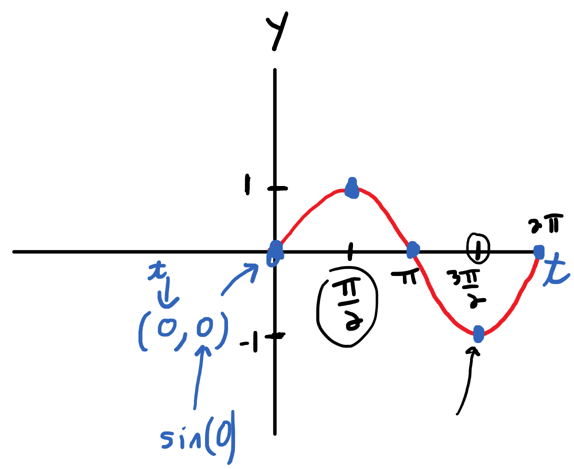


[Unit Circle and Sine Wave]

MrLovellFord

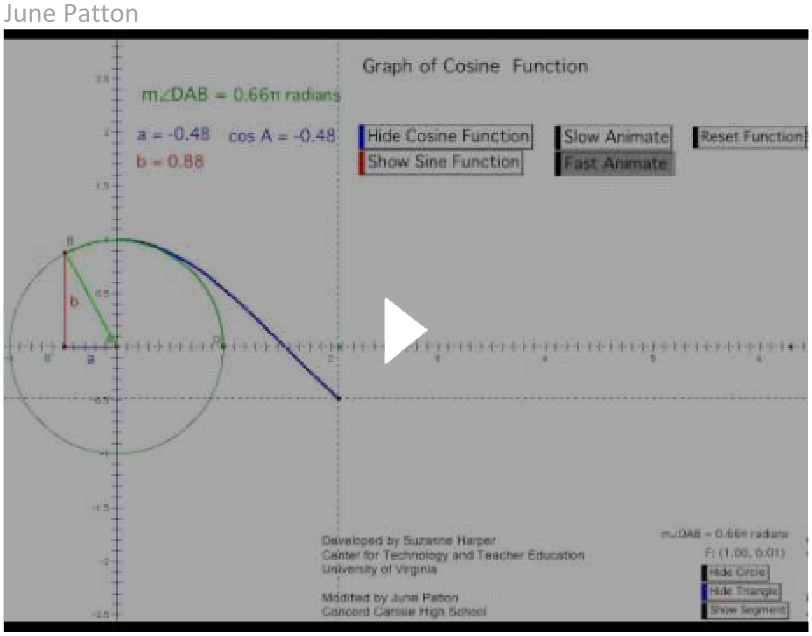


$y = \sin(t)$

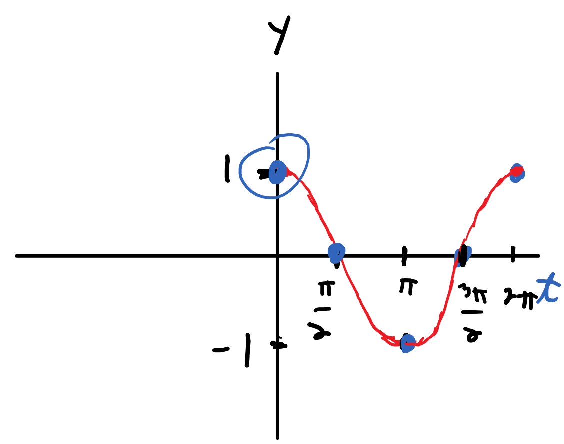


[Cosine Function from Unit Circle]

June Patton

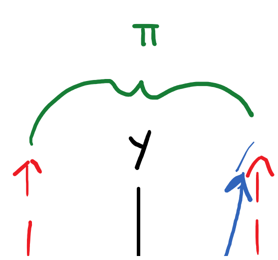


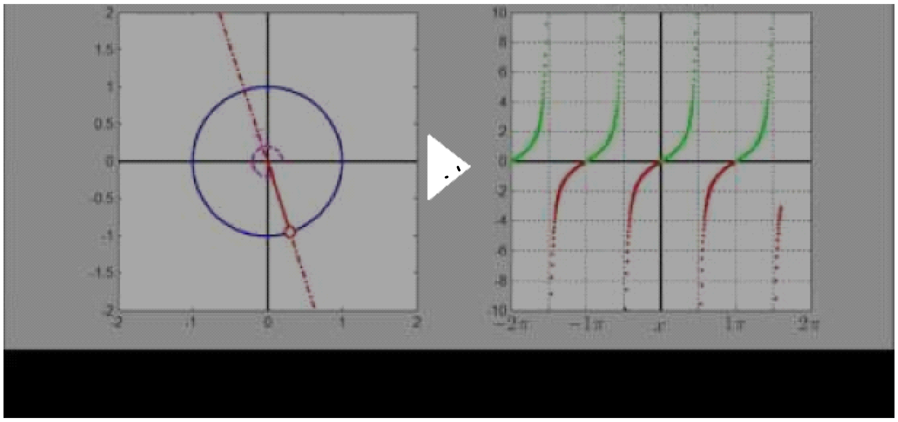
$y = \cos t$

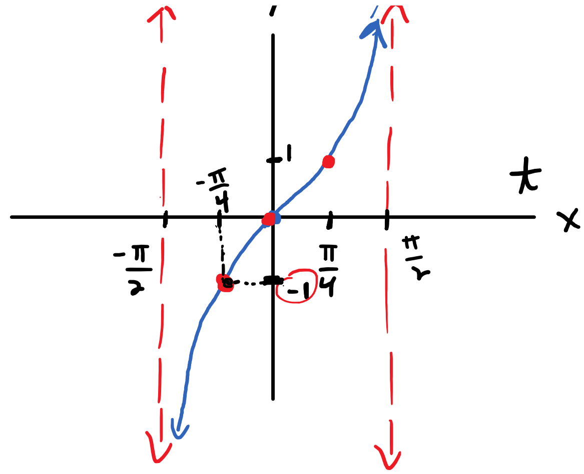


[What does the Tangent graph look like?]

Jonathan Mitchell



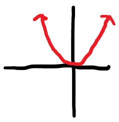




Even/odd Functions

let $y = f(x)$

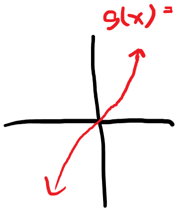
$f(x)=x^2$



sym. to y-axis

① $f(-x) = f(x)$ ^ f is even

$g(x)=x^3$



sym. to origin

② $f(-x) = -f(x)$ ^ f is odd

Is $f(x)$ even, odd, or neither?

ex $f(x) = x^2$

$f(-x) = (-x)^2$
$= x^2$
$= f(x)$

so f is even

ex Is $f(x)$ even, odd, or neither?

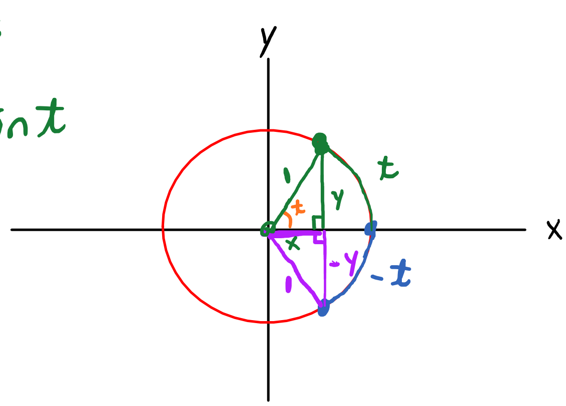
$g(x) = x^3$

$g(-x) = (-x)^3 = -x^3 = -g(x)$ so g is odd.

Note: cos t and sec t are ___even___. The rest are ___odd___.

$\cos(-t) = \cos t$

$\sin(-t) = -\sin t$



$\cos t = x$

$\cos(-t) = x$

So $\cos t = \cos(-t)$

which means cosine is an even fctn.

Example: Is it even, odd, or neither? Note that x is the input here.

$$\boxed{f(x) = \frac{\cos x}{x}}$$

$$f(-x) = \frac{\cos(-x)}{(-x)}$$

$$= \frac{\cos x}{-x}$$

$$= -\frac{\cos x}{x}$$

$$= -f(x).$$ so f is odd.

$$\frac{2}{-3} = -\frac{2}{3} \checkmark$$

Notes: Let $y = f(t)$.

1. A function is **periodic** if there is a smallest number p such that $f(t+p) = f(t)$.
2. The period of sine, cosine, secant, and cosecant is $360^\circ = \boxed{2\pi}$.
3. The period of tangent and cotangent is $180^\circ = \pi$.

$\sin(370^\circ) = \sin(10^\circ)$

$\sin(10^\circ + 360^\circ) = \sin(10^\circ)$

Fundamental Identities

Reciprocal: $\sin t = \frac{1}{\csc t}$, $\cos t = \frac{1}{\sec t}$, $\tan t = \frac{1}{\cot t}$

Ratio: $\tan t = \frac{\sin t}{\cos t}$, $\cot t = \frac{\cos t}{\sin t}$

Pythagorean: $\cos^2 t + \sin^2 t = 1$ $\quad 1 + \tan^2 t = \sec^2 t$

$\sin^2 t = 1 - \cos^2 t$ $\quad 1 + \cot^2 t = \csc^2 t$

$\cos^2 t = 1 - \sin^2 t$

Example: Write as a single trigonometric function: $\frac{1 - \sin^2 t}{\cos^2 t}$

$\sin^2 t = (\sin t)^2$

Example: Write as a single trigonometric function: $\dfrac{1-\sin^2 t}{\cot^2 t}$

$\sin^2 t = (\sin t)^2$

$\sin(t)^2$

$$= \frac{\cos^2 t}{\cot^2 t}$$

$$= \frac{\cos^2 t}{\left(\dfrac{\cos t}{\sin t}\right)^2}$$

$$= \frac{\cos^2 t}{\dfrac{\cos^2 t}{\sin^2 t}}$$

$$= \cancel{\cos^2 t} \cdot \frac{\sin^2 t}{\cancel{\cos^2 t}}$$

$$= \sin^2 t$$