Pythagorean Theorem 1

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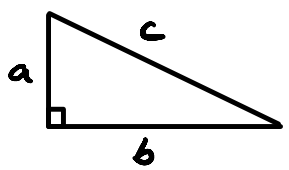
10:18 PM

Goals:

1. To find the length of a missing side of a right triangle.
2. To find the supplement and complement of a given angle.
3. To find a coterminal angle.
4. To set up a function involving the Pythagorean Theorem.

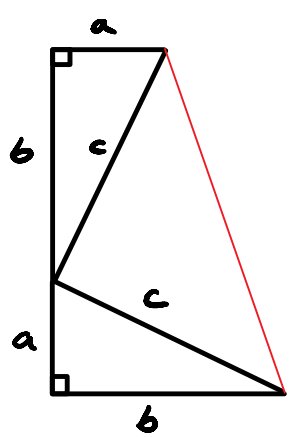
The Pythagorean Theorem

Given $\longrightarrow$



conclusion: $a^2 + b^2 = c^2$

Proof: (By President Garfield)



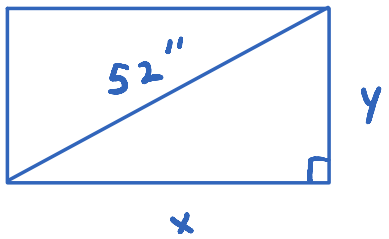
Note that the figure is a trapezoid (why?). Its area is given by the sum of the component triangles as well as the formula for area of a trapezoid.

$$\frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2 = \overbrace{\frac{1}{2}(a+b)(a+b)}^{\frac{1}{2}h(b_1+b_2)}$$

$$\cancel{ab} + \frac{1}{2}c^2 = \frac{1}{2}a^2 + \cancel{ab} + \frac{1}{2}b^2$$

$$c^2 = a^2 + b^2$$

Ex. A TV has an of aspect ratio 16 inches by 9 inches and a 52 inch diagonal. Find the dimensions.



$$x^2 + y^2 = 52^2$$

$$\frac{x}{y} = \frac{16}{9}$$

$$x = \frac{16}{9}y$$

$$\left(\frac{16}{9}y\right)^2 + y^2 = 52^2$$

$$\frac{256}{81}y^2 + y^2 = 52^2$$

$$\frac{337}{81}y^2 = 2704$$

$$y^2 = 2704 \cdot \frac{81}{337}$$

$$y^2 = \frac{219024}{337}$$

$$y = \sqrt{\frac{219024}{337}} \text{ inches}$$

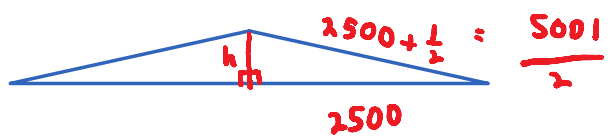
$$\approx 25.5 \text{ inches}$$

$$x = \frac{16}{9} \cdot \sqrt{219024} \text{ inches}$$

$$x = \frac{16}{9}\sqrt{\frac{219024}{337}} \text{ inches}$$

$$\approx 45.3 \text{ inches}$$

Ex. A bridge that is 5000 feet long expands 1 foot in the summer heat. For simplicity, assume the bridge bows up in the middle. Use the Pythagorean Theorem to estimate the height of the bridge midway across it.



Note that the altitude, h, represents how much the bridge bows in the middle

$$2500^2 + h^2 = \left(\frac{5001}{2}\right)^2$$

$$h^2 = \left(\frac{5001}{2}\right)^2 - 2500^2$$

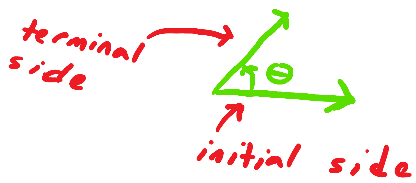
$$h^2 = \frac{10001}{4}$$

$$h = \frac{\sqrt{10001}}{2} \approx 50 \text{ ft}$$

The bridge bulges up about 50 feet!

<u>Def</u>: ① An <u>angle</u> is formed by rotating a ray about its end point.

a ray about its end point.



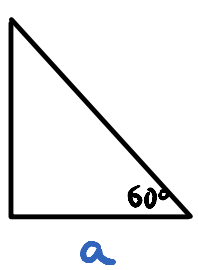
② 1 degree is $\frac{1}{360}$ of a full rotation

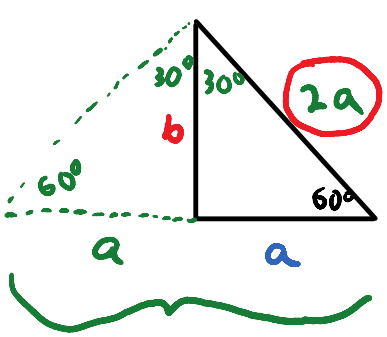
③ coterminal angles share the same terminal side

ex Find a coterminal angle for 790° that is between 0° and 360°

$$790^\circ - 360^\circ - 360^\circ = 70^\circ$$

ex Find the exact length of the missing side.





$2a$

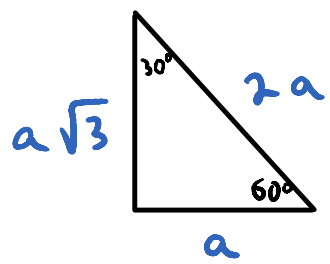
$$a^2 + b^2 = (2a)^2$$

$$b^2 = 4a^2 - a^2$$

$$\sqrt{b^2} = \sqrt{3a^2}$$

$$b = \sqrt{3}\, a$$

<u>Note</u>: In a $30^\circ - 60^\circ - 90^\circ$ Triangle, the sides are...



assumes a is the shortest side, opposite the 30° angle

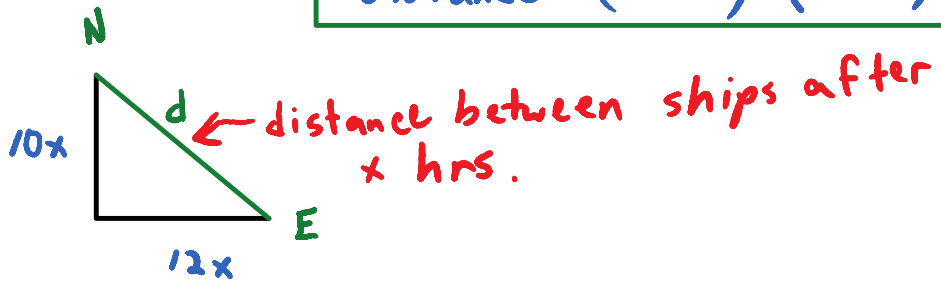
informal

<u>Def</u>: A <u>function</u> is an equation of the form $y =$ (some formula in terms of x) as long as any particular x-value yields a single y-value.

ex Two ships leave the same port at the same time, one is traveling north at 10 mph, and the other goes east at 12 mph.

Write a function that gives the distance between the ships after x hours.

$$\boxed{\text{Distance} = (\text{Rate}) \cdot (\text{Time})}$$



$$d^2 = (10x)^2 + (12x)^2$$

$$d^2 = 100x^2 + 144x^2$$

$$d^2 = 244x^2$$

$$d = \sqrt{244x^2}$$

$$d = 2\sqrt{61}\,x$$

Using function notation,

$d(x) = 2\sqrt{61}\,x$ (in miles)