Test 2 Practice

Wednesday, October 01, 2014
11:58 AM

1. Find the exact value of the following: $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

$\cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right]$ $\qquad \alpha = \frac{\pi}{4}$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\frac{3\pi}{4}$

2. Solve the following equation exactly for solutions in the interval $0 \le x < 2\pi$:

$2\sin x\cos x = \sqrt{3}\sin x$

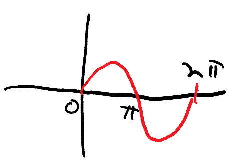
✓

$2\sin x\cos x - \sqrt{3}\sin x = 0$

$\sin x \cdot (2\cos x - \sqrt{3}) = 0$

$\sin x = 0$ or $2\cos x - \sqrt{3} = 0$

$x = 0, \pi$ $\qquad \cos x = \frac{\sqrt{3}}{2}$



$x = \frac{\pi}{6}$, $x = 2\pi - \frac{\pi}{6}$ $= \frac{11\pi}{6}$

$x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$

3. Solve the following equation for solutions in the interval $0 \le x < 2\pi$. Round your answers to the nearedst hundredth. DO NOT USE THE QUADRATIC FORMULA.

$2\tan^2 x - \tan x - 10 = 0$

$$2\tan^2 x - 1\tan x - 10 = 0$$

$$(2\tan x - 5)(\tan x + 2) = 0$$

$$\tan x = \frac{5}{2} \quad \text{or} \quad \tan x = -2$$

$$x = \tan^{-1}\left(\frac{5}{2}\right) \quad \text{or} \quad x = \tan^{-1}(-2)$$

$$x \approx 1.19 \qquad x \approx -1.11 + 2\pi = 5.18$$

$$\text{or } x \approx \pi + 1.19 = 4.33 \qquad \text{or } x \approx \pi - 1.11 = 2.03$$

4.

9. Solve the below equation using the quadratic formula for $0° \le x < 360°$. Round your answers to the nearest hundredth of a degree. (**Hint**: First, you will need to write $\csc x$ in terms of $\sin x$ and then clear the resulting fraction.) On practice test

$$3\sin x - 5 + \csc x = 0$$

$$\sin x\left[3\sin x - 5 + \frac{1}{\sin x} = 0\right]$$

$$3\sin^2 x - 5\sin x + 1 = 0$$

$$\sin x = \frac{5 \pm \sqrt{25 - 4(3)(1)}}{6}$$

$$\sin x = \frac{5 \pm \sqrt{13}}{6}$$

$$\sin x = \frac{5-\sqrt{13}}{6} \quad \text{or} \quad \cancel{\sin x = \frac{5+\sqrt{13}}{6}} > 1$$

$$x = \sin^{-1}\left(\frac{5-\sqrt{13}}{6}\right)$$

$$x \approx 13.44° \quad \text{or}$$

$$x \approx 166.56°$$

5. Solve the triangle with $b = 9.0$, $c = 14.0$ and angle $B = 32°$. Round answers to the nearest tenth.

$$\frac{14}{\sin C} = \frac{9}{\sin 32^\circ}$$

$$9 \sin C_1 = 14 \sin 32^\circ$$

$$\sin C_1 = \frac{14 \sin 32^\circ}{9}$$

case I, case II

$$C_1 \approx 55.5^\circ \text{ or } C_2 \approx 124.5^\circ$$

Case I: $C_1 \approx 55.5^\circ$

$$A = 180 - 55.5 - 32^\circ$$

$$= 92.5^\circ$$

$$\frac{a}{\sin 92.5} = \frac{9}{\sin 32^\circ}$$

$$a = \frac{9 \sin 92.5^\circ}{\sin 32^\circ}$$

$$a = \cancel{16}\ 17.0$$

case II

$$C_2 \approx 124.5^\circ$$

$$A \approx 180 - 124.5^\circ - 32$$

$$= 23.5^\circ$$

$$\frac{a}{\sin 23.5^\circ} = \frac{9}{\sin 32^\circ}$$

$$a = \frac{9 \sin 23.5^\circ}{\sin 32^\circ}$$

$$a \approx 6.8$$

6. Find the magnitude **and** direction of the vector $\vec{v} = \langle -2, -7 \rangle$. Round the measure of the direction angle to the nearest tenth of a degree.

$$\|\vec{v}\| = \sqrt{4 + 49} = \sqrt{53}$$

$$\tan \Theta = \frac{7}{2}$$

$$\alpha \approx 74.1^\circ$$

$$\Theta \approx 254.1^\circ$$

6.

Find the magnitude **and** direction of the vector $\vec{v} = <-2, -7>$. Round the measure of the direction angle to the nearest tenth of a degree.

$\|\vec{v}\| = \sqrt{4+49} = \sqrt{53}$

$\tan\theta = \frac{7}{2}$

$\alpha \approx 74.1^\circ$

$\theta \approx 254.1^\circ$

7.

Consider the vectors $\vec{u} = 4\vec{i} - 5\vec{j}$ and $\vec{v} = -2\vec{i} - 6\vec{j}$. Find $2\vec{u} - 3\vec{v}$ and write your answer in $\vec{i}, \vec{j}$ form.

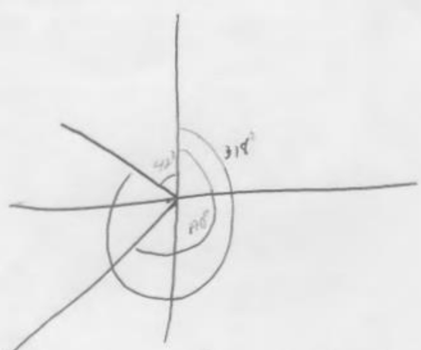
$2(4\vec{i} - 5\vec{j}) - 3(-2\vec{i} - 6\vec{j})$

$= 8\vec{i} - 10\vec{j} + 6\vec{i} + 18\vec{j}$

$= 14\vec{i} + 8\vec{j}$

8.

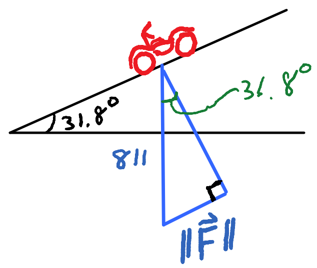
Two ships left a port at the same time. One ship traveled at a speed of 16 mph at a heading of 318°. The other ship traveled at a speed of 20 mph at a heading of 190°. Find the distance (to the nearest mile) between the two ships after 9 hours of travel. [**Hint**: Do **not** try to use vectors. USE THE LAW OF COSINES].



$c^2 = 144^2 + 180^2 - 2(144)(180)\cos 128$

$c \approx 291.6$ miles

9. A motorcycle that weighs 811 pounds is placed on a ramp that is inclined 31.8°. Find the magnitude of the force needed to keep the motorcycle from rolling down the ramp. Round to the nearest tenth of a pound.



$$\sin(31.8°) = \frac{\|\vec{F}\|}{811}$$

$$\|\vec{F}\| = 811\sin(31.8°)$$
$$= 427.4 \text{ lb}$$

10. Multiply and simplify: $(3-7i)(7-3i)$

$-58i$

11. Find the quotient $10cis(75°) \div 2cis(15°)$. Write the exact answer in standard form.

$\frac{10}{2} cis(75° - 15°)$

$5 cis\ 60°$

$5(\cos 60° + i \sin 60°)$

$5\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\frac{5}{2} + \frac{5i\sqrt{3}}{2}$

12. Find $[\sqrt{2}(\cos 10° + i\sin 10°)]^{10}$ using Demoivre's Theorem for powers. Leave your answer in trigonometric form.

$(\sqrt{2})^{10} cis\ 100°$

$32\ cis\ 100°$

Angle Betwee vectors $\vec{u}, \vec{v}$

$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}. \quad \text{Thus,}$$

$$\vec{u} \bullet \vec{v} = \underbrace{\|\vec{u}\|\,\|\vec{v}\|\cos\theta}_{=0 \text{ if } \theta = 90°}$$

So, If $\vec{u} \bullet \vec{v} = 0$, then $\vec{u} \perp \vec{v}$

(ex) Find $\vec{u} \bullet \vec{v}$ where $\vec{u} = \langle 2, 3 \rangle$
and $\vec{v} = \langle -3, -2 \rangle$

$\vec{u} \bullet \vec{v} = -6 + 6 = 0$ ← answer

So $\vec{u} \perp \vec{v}$.