17 on 11.9

Monday, May 05, 2014
12:31 PM

11.9

17 $\quad f(x) = \dfrac{x}{(1+4x)^2}$

$$\boxed{\left(\frac{1}{u}\right)' = -\frac{1}{u^2}}$$

consider $\dfrac{1}{(1+4x)^2}$

$$\frac{1}{1+4x} = (1+4x)^{-1}$$

$$\left[(1+4x)^{-1}\right]' = -4(1+4x)^{-2}$$

So $\left[-\frac{1}{4} \cdot \frac{1}{1+4x}\right]' = \dfrac{1}{(1+4x)^2}$

$\left[\ \ 4 \quad \underbrace{1+4x}\ \right] \qquad (1+4x)$

get a power series for this

$$\frac{1}{1+4x} = \frac{1}{1-(-4x)}$$

$$= \sum_{n=0}^{\infty} (-4x)^n$$

so $$-\frac{1}{4}\cdot\frac{1}{1+4x} = -\frac{1}{4}\sum_{n=0}^{\infty}(-4x)^n$$

$$\frac{1}{(1+4x)^2} = \left[-\frac{1}{4}\sum_{n=0}^{\infty}(-4x)^n\right]', \qquad -1<-4x<1$$

$$-\frac{1}{4}<x<\frac{1}{4}$$

$$R=\frac{1}{4}$$

$$= \left[-\frac{1}{4}\sum_{n=0}(-1)^n 4^n x^n\right]'$$

$$= -\frac{1}{4} \sum_{n=1}^{\infty} (-1)^n 4^n n x^{n-1}$$

$$\frac{x}{(1+4x)^2} = -\frac{1}{4^1} \sum_{n=1}^{\infty} (-1)^n 4^n n x^n$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} 4^{n-1} n x^n$$

$$= \sum_{n=0}^{\infty} (-1)^n 4^n (n+1) x^{n+1}, \quad R = \frac{1}{4}$$