6. Find the arc length of the graph of $f(x) = \frac{2}{3}x^{3/2} + 1$ on the interval [1, 3]. Please leave your answer in exact form using only fractions (i.e. no decimals allowed!).

$$\frac{2}{3}\left[8 - 2\sqrt{2}\right]$$

7. Set up an integral that represents the area of the surface of revolution generated by revolving the following curve about the y-axis: $f(x) = \sqrt[3]{x} + 2$, $1 \le x \le 8$. (**Note: set up the integral only**, DO NOT EVALUATE THE INTEGRAL!!! However, you should find any derivative required by the area formula and simplify the integrand)

$$S = 2\pi \int_1^8 t \sqrt{\frac{1}{9t^{4/3}} + 1}\, dt$$

(assuming $x = t$ and $y = \sqrt[3]{t} + 2$ for $1 \le t \le 8$)

8. Consider the parametric equations $x = 2\sin t$, $y = 3\cos t$ where $0 \le t \le 2\pi$.

a) Eliminate the parameter to find a Cartesian equation of the curve.

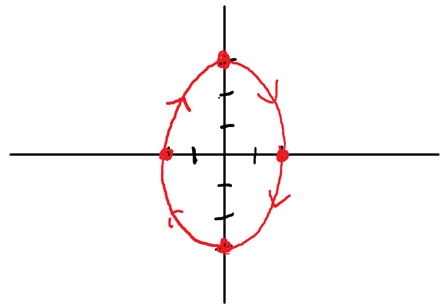
$\frac{x}{2} = \sin t$, $\frac{y}{3} = \cos t$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

b) Graph the curve.

ellipse

center: (0, 0)



c) describe the motion (and direction) of a particle with position (x, y) on the curve as t goes from 0 to 2π.

start point

t	x	y
0	0	3
$\frac{\pi}{2}$	2	0

goes clockwise

It goes clockwise once around the ellipse starting at (0, 3).

9. a) Consider the parametric equations $x = t^3 - 3t^2$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ in terms of t.

$$\frac{dy}{dx} = \frac{t^2-1}{t^2-2t}$$

b) Find the points where the tangent line is vertical.

$(0,0)$ $(-4,2)$

10. Use the parametric formula to find the area of the surface of revolution generated by revolving the following curve about the y-axis: $x = t$, $y = 2t$ where $0 \le t \le 4$.

$16\pi\sqrt{5}$

11. Convert to a Cartesian equation and identify the curve by writing it in standard form: $r = 3\cos\theta$

$$\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$$

Circle w/ center $\left(\frac{3}{2}, 0\right)$

and radius = $\frac{3}{2}$

12. Find the points of horizontal tangency to the polar curve $r = 1 + \sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$x = (1+\sin\theta)\cos\theta$ | $y = (1+\sin\theta)\sin\theta$

$\boxed{x = \cos\theta + \sin\theta\cos\theta \quad | \quad y = \sin\theta + \sin^2\theta}$

parametric eqn

$\boxed{\text{H-tan}}$ set $\frac{dy}{d\theta} = 0$

$$\frac{dy}{d\theta} = \boxed{\cos\theta + 2\sin\theta\cos\theta = 0}$$

$$\cos\theta(1 + 2\sin\theta) = 0$$

$\cos\theta = 0$ or $1 + 2\sin\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin\theta = -\frac{1}{2}$ $\quad$ $\left(\alpha = \frac{\pi}{6}\right)$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\frac{3\pi}{2}$

$r = 1 + \sin\theta$

$r = 1 + \sin\theta$ $\frac{3\pi}{2}$

$\boxed{(2, \frac{\pi}{2}) \quad \cancel{(0, \frac{3\pi}{2})} \quad (\frac{1}{2}, \frac{7\pi}{6}) \quad (\frac{1}{2}, \frac{11\pi}{6})}$ Answer

$x = \cos\theta + \sin\theta\cos\theta$

$$\frac{dx}{d\theta} = \boxed{-\sin\theta + \underbrace{\cos^2\theta}_{1-\sin^2\theta} - \sin^2\theta = 0}$$

$$-\sin\theta + 1 - 2\sin^2\theta = 0$$

$$2\sin^2\theta + \sin\theta - 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = -1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \qquad \theta = \frac{3\pi}{2}$$

has a vertical tangent at $(0, \frac{3\pi}{2})$ (see graph)

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{3\pi}{2}} = \frac{0}{0} \quad \text{problem}$$

13. Find the area enlcosed by the inner loop of $r = 1 + 2\cos\theta$.