Interpreting Integrals by their Differentials and Other Musings

Thursday, May 7, 2015 3:03 PM

$\int_a^b f(x)\,dx$ — dx: tiny length along x-axis (x-axis element)

$\iint_R f(x,y)\,dA$ — dA: area element ($dy\,dx$, $r\,dr\,d\theta$)

$\iiint_Q f(x,y,z)\,dV$ — dV: volume element ($dz\,dy\,dx$ (5 other orders), $r\,dz\,dr\,d\theta$, $\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$)

$\iint_R \rho(x,y)\,dA$ — mass element, $\iiint_Q \rho(x,y,z)\,dV$ — mass element | ρ is a density function

$\int_C f(x,y,z)\,ds$ — ds: arc length element

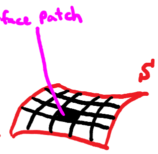
$\iint_S f(x,y,z)\,dS$ — dS: surface area element

$\iint_S \vec{F}\cdot\vec{N}\,dS$ — Flux element (through surface patch)

Stokes': $\int_C \vec{F}\cdot\vec{T}\,ds = \iint_S (\text{curl}\,\vec{F})\cdot\vec{N}\,dS$

$\vec{F}\cdot\vec{T}\,ds$ ($= \int_C \vec{F}\cdot d\vec{r}$): circulation-element (on sub-arc of C)

$(\text{curl}\,\vec{F})\cdot\vec{N}\,dS$: circulation element (around surface patch)



Divergence: $\iint_S \vec{F}\cdot\vec{N}\,dS = \iiint_E \text{div}\,\vec{F}\,dV$

$\vec{F}\cdot\vec{N}\,dS$: Flux element (through a surface patch)

$\text{div}\,\vec{F}\,dV$: Flux element (through point in E, since $\text{div}\,F(P)$ = flux density)

$\text{div}\,\vec{F}(P) = \dfrac{\text{Flux}}{\text{volume}}$ — flux density

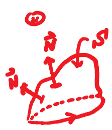
Major Theorems (Higher Dimensional Analogs of FTC)

FTC: $\int_a^b F'(x)\,dx = F(b) - F(a)$

FTLI: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

Green: $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_{C_{closed}} \vec{F}\cdot d\vec{r}$

Stokes': $\iint_S (\text{curl}\,\vec{F})\cdot\vec{N}\,dS = \int_C \vec{F}\cdot d\vec{r}$



Divergence: $\iiint_E \text{div}\,\vec{F}\,dV = \iint_S \vec{F}\cdot\vec{N}\,dS$