Section 14.3: Partial Derivatives

Tuesday, February 24, 2015 4:40 PM

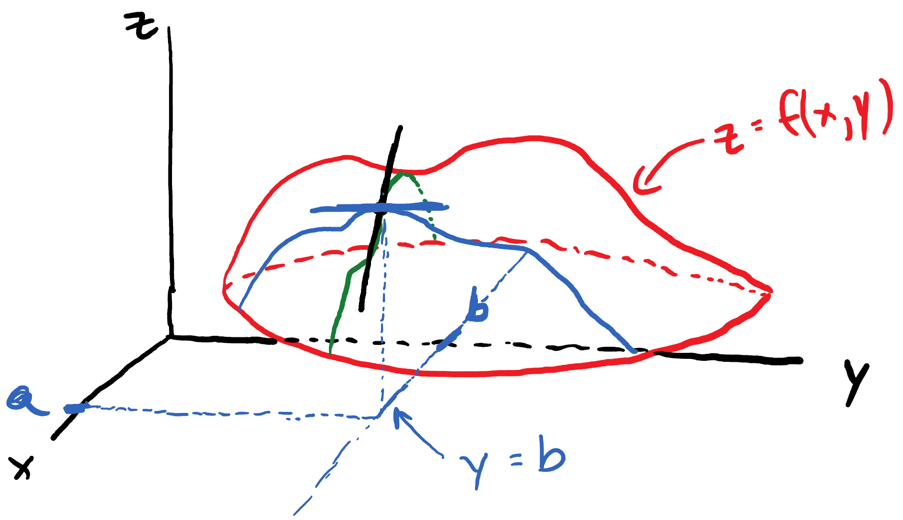
Goal: To calculate and interpret partial derivatives

Def: Let $z = f(x,y)$. The partial derivatives w.r.t x and y are ....

$$\boxed{f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}}$$ holding y constant

$$f_y(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

$z = f(x,y)$ graphs to be a surface



$f_y(a,b)$

$\boxed{f_x(a,b)}$ slope of the line in black at $(a, b, f(a,b))$

ex Let $f(x,y) = x^2 - 2xy + y^2$. Find f_x using definition.

definition.

$$f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x)y + y^2 - (x^2 - 2xy + y^2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - 2\cancel{x}y - 2\Delta x y + \cancel{y^2} - \cancel{x^2} + 2\cancel{x}y - \cancel{y^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x - 2y)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x - 2y) = 2x - 2y$$

Example : The temperature-humidity index I gives the perceived air temperature as a function of the actual temperature is T and the relative humidity is h. So I can be written as $I = f(T,h)$. Estimate the value of $f_T(95,70)$. What is the practical interpretation of this value?

T h

Actual Temperature (°F) T \ h	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

$$I = f(T, h)$$

$$f_T(T,h) \approx \frac{f(T+\Delta T, h) - f(T,h)}{\Delta T}$$

ΔT

$\boxed{\text{Let } \Delta T = 5}$

$$f_T(95,70) \approx \frac{f(95+5,70) - f(95,70)}{5}$$

$$= \frac{f(100,70) - f(95,70)}{5}$$

$$= \frac{144 - 124}{5}$$

$$= \frac{20}{5}$$

$$= 4$$

$\boxed{\Delta T = -5}$ $\quad f_T(T,h) \approx \dfrac{f(T+\Delta T,h) - f(T,h)}{\Delta T}$

$$f_T(95,70) = \frac{f(90,70) - f(95,70)}{-5}$$

$$= \frac{106 - 124}{-5}$$

$$= \frac{-18}{-5}$$

$$= 3.6$$

we'll take the average as our estimate

$\frac{\partial f}{\partial T}$

$\frac{\text{perceived air temp}}{\text{actual temp}}$

$$f_T(95,70) \approx \frac{4+3.6}{2} = \frac{3.8}{1}$$

So, when the actual temp is $95°$ and the humidity is 70%, for every $1°$ increase in actual temp, the perceived air temp is increasing by $3.8°$.

ex Let $h(x,y) = \boxed{x^2-y^2}$. Find the slopes of the surface in x and y directions at $(-2,1,3)$.

$$h_x(x,y) = 2x \longrightarrow h(-2,1) = 2(-2) = -4$$

$$h_y(x,y) = -2y \rightarrow h(-2,1) = -2(1) = -2$$

<u>Notation</u>: $z = f(x,y)$

$$f_x(x,y) = \frac{\partial}{\partial x} f(x,y) = \frac{\partial f}{\partial x} = z_x$$

$$f_y(x,y) = \frac{\partial}{\partial y} f(x,y) = \frac{\partial f}{\partial y} = z_y$$

$$f_y(x,y) = \frac{\partial}{\partial y} f(x,y) = \frac{\partial f}{\partial y} = z_y$$

Theorem: f_{xy} and f_{yx} are continuous on an open disk in $\mathbb{R}^2$, then for every (x,y) in the disk....

$$\boxed{f_{xy}(x,y) = f_{yx}(x,y)}$$ mixed partials equal

ex Find the four partial derivatives of

$z = 2xe^y - 3ye^{-x}$

$z_x = 2e^y + 3ye^{-x}$, $z_y = 2xe^y - 3e^{-x}$

$z_{xx} = -3ye^{-x}$, $z_{yy} = 2xe^y$

$z_{xy} = 2e^y + 3e^{-x}$, $z_{yx} = 2e^y + 3e^{-x}$

Note: $z_{xy} = z_{yx}$

ex Find $\frac{\partial z}{\partial x}$ using implicit differentiation

$$x^2+y^2+z^2 = \boxed{3xyz} \rightarrow 3y\,(xz)$$

$$2x + 0 + 2z \cdot \frac{\partial z}{\partial x} = 3y\left(z + x\frac{\partial z}{\partial x}\right)$$

$$2x + 2z\frac{\partial z}{\partial x} = 3yz + 3xy\frac{\partial z}{\partial x}$$

$$2z\frac{\partial z}{\partial x} - 3xy\frac{\partial z}{\partial x} = 3yz - 2x$$

$$(2z - 3xy)\frac{\partial z}{\partial x} = 3yz - 2x$$

$$\frac{\partial z}{\partial x} = \frac{3yz - 2x}{2z - 3xy}$$

ex Let $f(r,s,t) = \boxed{r\ln(rs^2t^3)}$. Find $\dfrac{\partial^3 f}{\partial s^2\,\partial r}$

$$\frac{\partial^3 f}{\partial s^2\,\partial r} = \frac{\partial}{\partial s}\left(\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial r}\right)\right)$$

$$\frac{\partial f}{\partial r} = \ln(rs^2t^3) + \cancel{r}\cdot\frac{1}{\cancel{r}\cancel{s^2}\cancel{t^3}}\cdot\cancel{s^2}\cancel{t^3}$$

$$\frac{\partial^2 f}{} = \ln(rs^2t^3) + 1$$

$$\frac{\partial^2 f}{\partial s\,\partial r} = \ln(rs^2t^3) + 1$$

$$\frac{\partial}{\partial s}\cdot\frac{\partial f}{\partial r} = \frac{1}{\cancel{r}\,s^{\cancel{2}}\,\cancel{t^3}}\cdot 2\cancel{r}\cancel{s}\cancel{t^3} = \frac{2}{s}$$

$$\frac{\partial}{\partial s}\left(\frac{\partial^2 f}{\partial s\,\partial r}\right) = -\frac{2}{s^2}$$