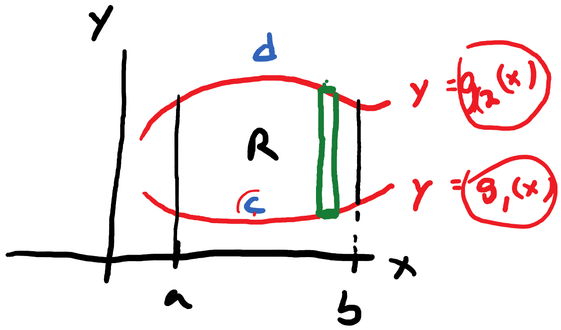
Section 15.3: More Fun with Double Integrals

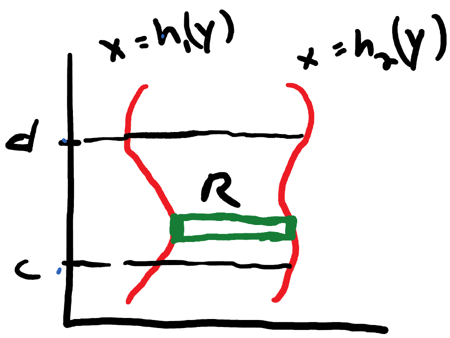
Tuesday, March 24, 2015 5:02 PM

Def: (in pictures)



Type I Region

$$\iint_R f(x,y)\,\underbrace{dy\,dx}_{dA} = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$



Type II

$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

Note: If $f(x,y) = 1$, $\iint_R f(x,y)\,dA$ = Area of R

$$\iint_R f(x,y)\,dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} \overset{y}{1}\,dy \right] dx$$

$$= \int^b \left[y \right]_{g_1(x)}^{g_2(x)} dx$$

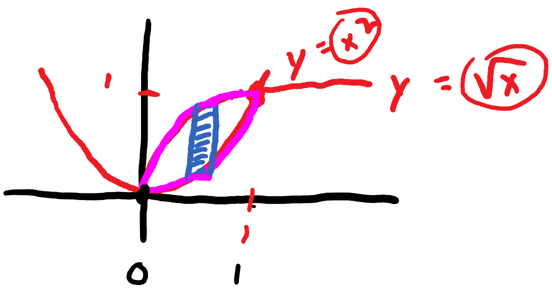
$$= \int_a^b \left[y\right]_{g_1(x)}^{g_2(x)} dx$$

$$= \int_a^b (g_2(x) - g_1(x))\, dx$$

$$= \text{Area } R$$

ex Evaluate $\iint_R \overset{f}{(x+y)}\, dA$, R bounded by

$y = \sqrt{x}$ and $y = x^2$.



$$\iint_R (x+y)\, dA = \int_0^1 \int_{x^2}^{\sqrt{x}} (x+y)\, dy\, dx$$

$$\int_0^1 \left[\int_{x^2}^{\sqrt{x}} (x+y)\, dy\right] dx$$

$$\int_0^1 \left[xy + \frac{1}{2}y^2\right]_{y=x^2}^{y=\sqrt{x}} dx$$

$$\int_0^1 \left[\left(x^{3/2} + \frac{1}{2}x\right) - \left(x^3 + \frac{1}{2}x^4\right)\right] dx$$

$$\left[\frac{2}{5}x^{5/2} + \frac{1}{4}x^2 - \frac{1}{4}x^4 - \frac{1}{10}x^5\right]_0^1$$

$\frac{2}{5} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} - \frac{1}{10}$

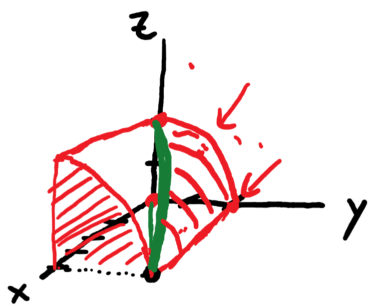
$\frac{4}{10} - \frac{1}{10}$

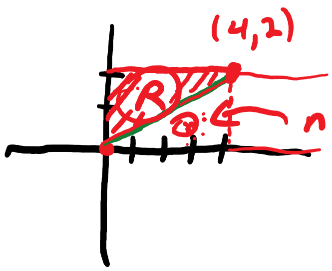
$\frac{3}{10}$

ex Find volume of the solid bounded by the cylinder $y^2 + z^2 = 4$, plane $x = 2y$ and in the first octant.

solve for z to get $f(x,y)$

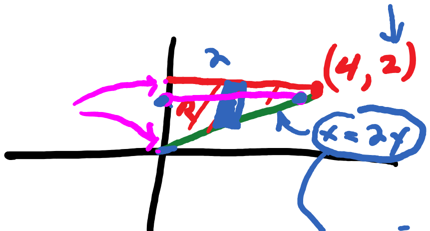
$z = \underbrace{\sqrt{4 - y^2}}_{\text{integrand}}$





not ~~closed~~

bounded on right



$y = \frac{x}{2}$

$$V = \int_0^4 \left[\int_{\frac{x}{2}}^{2} \sqrt{4-y^2}\, dy \right] dx$$

$$V = \int_0^2 \left[\int_0^{2y} \sqrt{4-y^2}\, dx \right] dy$$

$$V = \int_0^2 \left[x \sqrt{4-y^2} \right]_{x=0}^{x=2y} dy$$

$$V = \int_0^2 2y\,\sqrt{\underbrace{4-y^2}_{u}}\;dy \qquad u,\; du = -2y\,dy$$

$$V = -\int_0^2 \overbrace{\sqrt{4-y^2}}^{\sqrt{u}}\;\overbrace{(-2y)\,dy}^{du}$$

$$V = -\left[\frac{2}{3}(4-y^2)^{\frac{3}{2}}\right]_0^2$$

$$= -\frac{2}{3}\left[(4-y^2)^{\frac{3}{2}}\right]_0^2$$

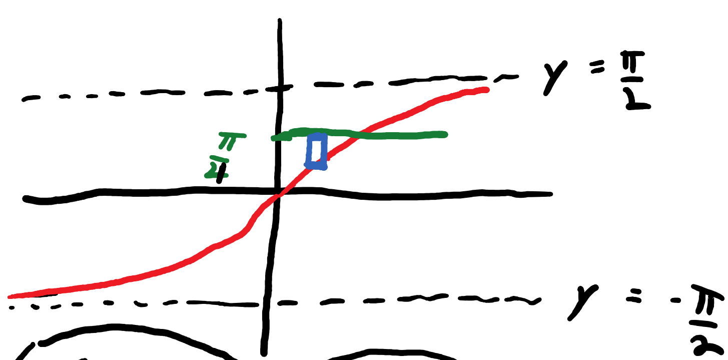
$$= -\frac{2}{3}\left[0 - 4^{\frac{3}{2}}\right]$$

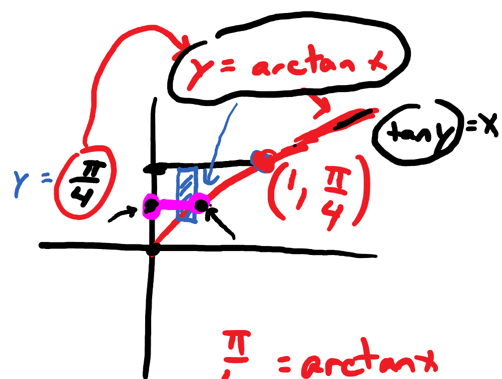
$$= \frac{16}{3}$$

ex Reverse the order of integration.

$$\int_0^1 \int_{\arctan x}^{\frac{\pi}{4}} f(x,y)\,dy\,dx$$

$$\arctan x \le y \le \frac{\pi}{4}$$
$$0 \le x \le 1$$





$$\frac{\pi}{4} = \arctan x$$

$$\tan\frac{\pi}{4} = x$$

$$\int^{\frac{\pi}{4}} \int^{\tan y} f(x,y)\,dx\,dy$$

$$\int_0^{\frac{\pi}{4}} \int_0^{\tan y} f(x,y)\, dx\, dy$$

$\tan \frac{\pi}{4} = x$