Section 16.1: Vector Fields

Thursday, April 16, 2015 3:30 PM

Goals:

1. To sketch a vector field.
2. To find the gradient vector field of a function.

Definition: A vector field, **F**, is a function that maps points to vectors.

plane $\longrightarrow$ $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j} = \langle P, Q \rangle$

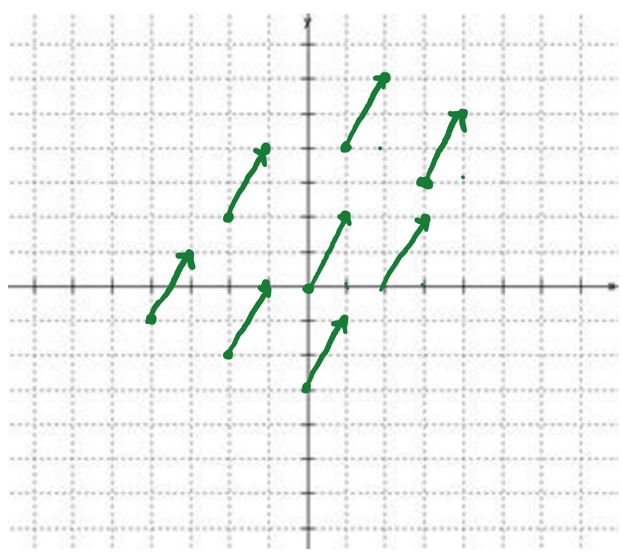
space $\longrightarrow$ $\vec{F}(x,y,z) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$

$= \langle P, Q, R \rangle$

ex sketch $\vec{F}$

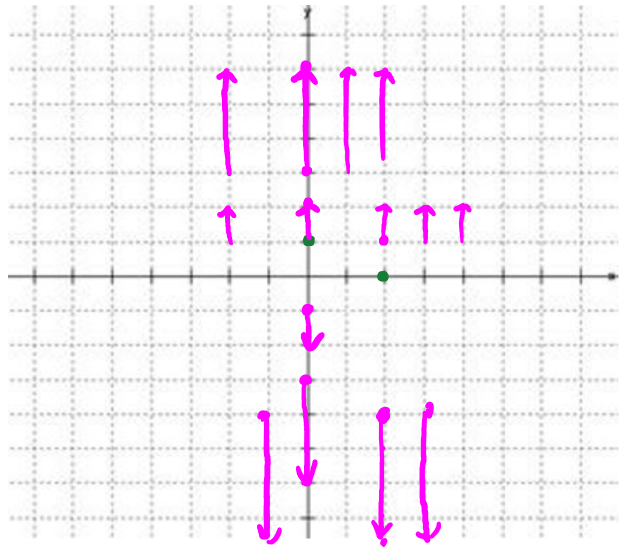
a) $\vec{F}(x,y) = 1\vec{i} + 2\vec{j} = \langle 1, 2 \rangle$

input is the tail.



b) $\vec{F}(x,y) = y\,\vec{j} = \langle 0, y \rangle$

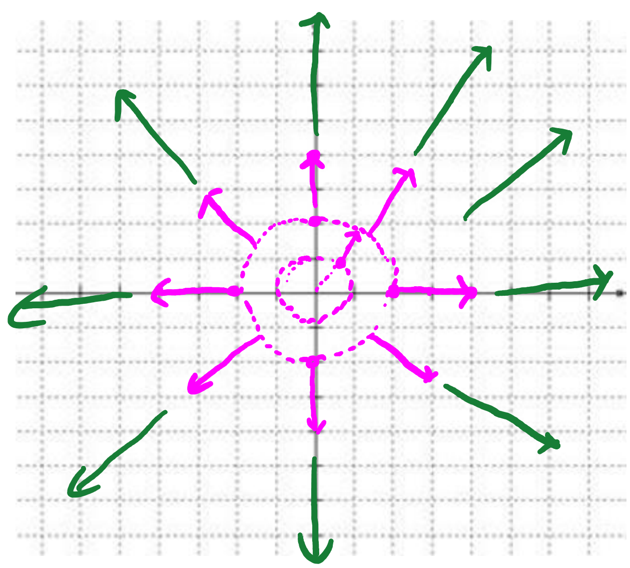
input	output
(x,y)	$\langle 0,y\rangle$
$(2,0)$	$\langle 0,0\rangle$
$(0,1)$	$\langle 0,1\rangle$
$(0,3)$	$\langle 0,3\rangle$
$(0,-1)$	$\langle 0,-1\rangle$



c) $\vec{F}(x,y) = x\vec{i} + y\vec{j} = \langle x,y\rangle$

Each vector points radially out from origin.

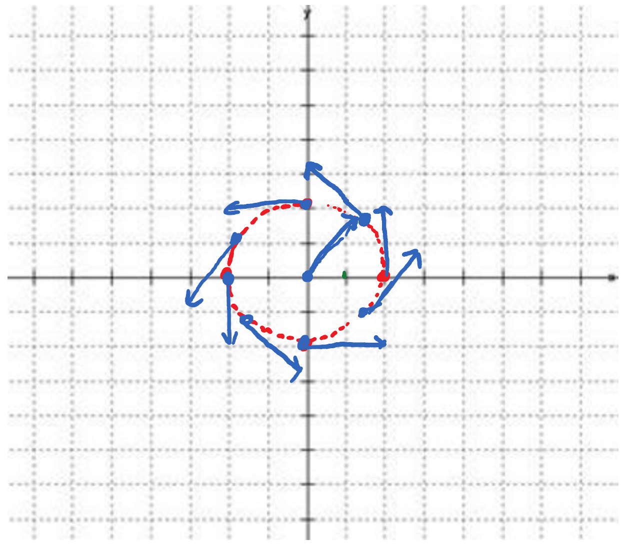
$\|\vec{F}\| =$ distance from origin



d) $\vec{F}(x,y) = -y\vec{i} + x\vec{j} = \langle -y, x\rangle$

rotates $\langle x,y\rangle$ by 90°

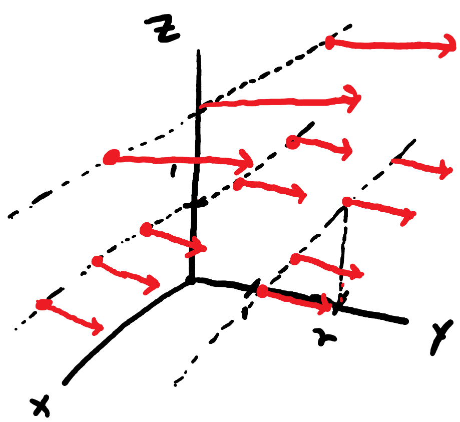
input	output
$(1,2)$	$\langle -2, 1 \rangle$



$$\|\vec{F}\| = \sqrt{x^2+y^2}$$

uniform
form motion
vector field

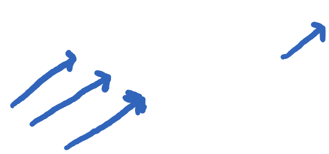
e) $F(x,y,z) = z\,\vec{j} = \langle 0, z, 0 \rangle$



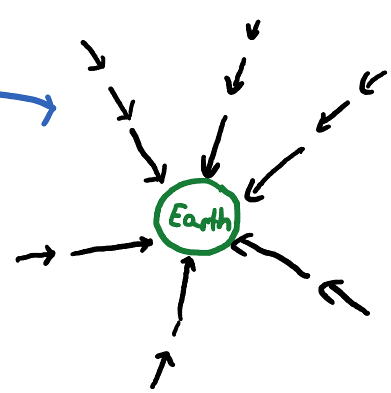
inputs	output
$(1, 3, 1)$	$\langle 0, 1, 0 \rangle$
$(5, -8, 1)$	
$(-10, 100, 1)$	

<u>Note</u>: Vector Fields are used to model...

① Weather (wind velocity)



$$\left.\begin{array}{l} \text{② gravitional field} \\ \text{③ magnetic field} \\ \text{④ electric field} \end{array}\right\} \text{force field}$$



④ electric field

Definition: A vector field, F, is conservative if there is a scalar function *f*(x,y,z) such that **F** = del f. $= \nabla f$ — gradient of f

$\nabla f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$

ex Find the gradient vector field of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$\nabla f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$$

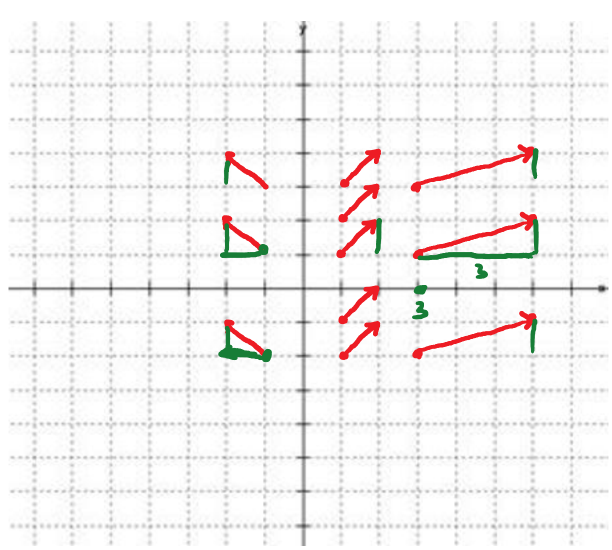
$$f_x(x,y,z) = \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{1}{2}} \cancel{2}x = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$f_y = \qquad = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$f_z = \qquad = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\boxed{\nabla f = \frac{x}{\sqrt{x^2+y^2+z^2}}\vec{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\vec{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}}$$

ex which vector field matches the plot.



a) $x\vec{i} - 2y\vec{j}$

b) $x\vec{i} + 1\vec{j}$

c) $1\vec{i} + y\vec{j} = \langle 1, y \rangle$