Warm-up: Computing TNB

Tuesday, February 10, 2015
5:00 PM

Warm-up

ex Let $\vec{r}(t) = t\,\vec{i} + \cos t\,\vec{j} + \sin t\,\vec{k}$

Find $\vec{T}(t) = \dfrac{r'(t)}{\|\vec{r}'(t)\|} = \dfrac{\vec{v}(t)}{\|\vec{v}(t)\|}$

$\vec{r}'(t) = 1\vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k}$

$\|\vec{r}'(t)\| = \sqrt{1 + \underbrace{\sin^2 t + \cos^2 t}} = \sqrt{2}$

$\vec{T}(t) = \dfrac{1}{\sqrt{2}}\left(1\vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k}\right)$

$\vec{N}(t) = \dfrac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$

$\vec{T}'(t) = \dfrac{1}{\sqrt{2}}\left(-\cos t\,\vec{j} - \sin t\,\vec{k}\right)$

$\|\vec{T}'(t)\| = \dfrac{1}{\sqrt{2}}\sqrt{\cos^2 t + \sin^2 t}$

$= \dfrac{1}{\sqrt{2}}$

$\vec{N}(t) = \dfrac{\cancel{\frac{1}{\sqrt{2}}}\left(-\cos t\,\vec{j} - \sin t\,\vec{k}\right)}{\cancel{\frac{1}{\sqrt{2}}}}$

$\vec{N}(t) = 0\vec{i} - \cos t\,\vec{j} - \sin t\,\vec{k}$

The unit Binormal vector $= \vec{B} = \vec{T} \times \vec{N}$

$$\vec{T}(t) = \frac{1}{\sqrt{2}}\left(1\vec{i} - \sin t\,\vec{j} + \cos t\,\vec{k}\right)$$

$c\vec{u} \times k\vec{v}$

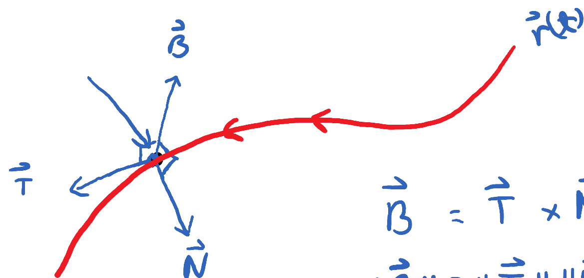
$= ck(\vec{u} \times \vec{v})$

$$\vec{B} = \vec{T} \times \vec{N}$$

$$= \frac{1}{\sqrt{2}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{vmatrix}$$

$$= \frac{1}{\sqrt{2}}\left[\left(\sin^2 t + (+\cos^2 t)\right)\vec{i} - \left(-\sin t\right)\vec{j} + \left(-\cos t\right)\vec{k}\right]$$

$$\vec{B}(t) = \frac{1}{\sqrt{2}}\left(\vec{i} + \sin t\,\vec{j} - \cos t\,\vec{k}\right)$$



TNB frame

$$\vec{B} = \vec{T} \times \vec{N}$$

$$\|\vec{B}\| = \underbrace{\|\vec{T}\|}\,\underbrace{\|\vec{N}\|}\,\underbrace{\sin 90^\circ}$$

$$\|\vec{B}\| = 1 \cdot 1 \cdot 1$$

$$\|\vec{B}\| = 1$$