Math 205 Final Exam Formula Sheet

1. Angle between two vectors: $\cos\theta = \dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\| \ \|\mathbf{v}\|}$

2. Projection of **u** onto **v**: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

3. Work formulas: $W = \left\|\text{proj}_{\overrightarrow{PQ}}\mathbf{F}\right\| \left\|\overrightarrow{PQ}\right\|$, $W = \|\mathbf{F}\| \left\|\overrightarrow{PQ}\right\|\cos\theta$, $W = \mathbf{F} \bullet \overrightarrow{PQ}$

4. Parametric equations of a line in space: $x = x_1 + at$, $y = y_1 + bt$, $z = z_1 + ct$

5. Equation of a plane: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

6. The distance between a plane and a point Q (not on the plane): $D = \left\|\text{proj}_{\mathbf{n}}\overrightarrow{PQ}\right\| = \dfrac{\left|\overrightarrow{PQ} \bullet \mathbf{n}\right|}{\|\mathbf{n}\|}$

7. The distance between a point Q and line in space is given by where **v** is the direction vector for the line and **P** is a point on the line: $D = \dfrac{\left\|\overrightarrow{PQ} \times \mathbf{v}\right\|}{\|\mathbf{v}\|}$.

8. Cylindrical to rectangular: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$

9. Rectangular to cylindrical: $r^2 = x^2 + y^2$, $\tan\theta = \dfrac{y}{x}$, $z = z$

10. Spherical to rectangular: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$

11. Rectangular to spherical: $\rho^2 = x^2 + y^2 + z^2$, $\tan\theta = \dfrac{y}{x}$, $\phi = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

12. Spherical to cylindrical: $r^2 = \rho^2\sin^2\phi$, $\theta = \theta$, $z = \rho\cos\phi$

13. Cylindrical to spherical: $\rho = \sqrt{r^2 + z^2}$, $\theta = \theta$, $\phi = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

14. The **unit tangent vector**: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \dfrac{\mathbf{v}(t)}{\|\mathbf{v}(t)\|}$.

15. The **principal unit normal vector**: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

16. $\mathbf{a}(t) = a_{\mathbf{T}}\ \mathbf{T}(t) + a_{\mathbf{N}}\ \mathbf{N}(t)$, where $a_{\mathbf{T}} = \dfrac{d}{dt}\|\mathbf{v}(t)\|$ and $a_{\mathbf{N}} = \|\mathbf{v}\|\,\|\mathbf{T}'(t)\|$

17. $a_{\mathbf{T}} = \mathbf{a} \bullet \mathbf{T}$

18. $a_{\mathbf{N}} = \mathbf{a} \bullet \mathbf{N} = \sqrt{\|\mathbf{a}\|^2 - a_{\mathbf{T}}^{\ 2}}$

19. Arc length formula: $s = \displaystyle\int_a^b \|\mathbf{r}'(t)\|dt$

20. Arc length function: $s(t) = \displaystyle\int_a^t \|\mathbf{r}'(u)\|du$

21. **Curvature**: $K = \|\mathbf{T}'(s)\| = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$ (incidentally, $K = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|r'(t)\|^3}$ is also a handy formula for calculating K).

22. $f_x(x,y) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$ and $f_y(x,y) = \lim_{\Delta y \to 0} \dfrac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$

23. The total differential: $dz = f_x(x,y)dx + f_y(x,y)dy$

24. Suppose that in the equation $F(x,y) = 0$, y is defined implicitly as a differentiable function of x. If F is differentiable, then $\dfrac{dy}{dx} = -\dfrac{F_x(x,y)}{F_y(x,y)}$.

25. If the equation $F(x,y,z) = 0$ defines z implicitly as a differentiable function of x and y, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x(x,y,z)}{F_z(x,y,z)}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{F_y(x,y,z)}{F_z(x,y,z)}$.

26. The directional derivative: $D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$.

27. The gradient: $\nabla f(x,y) = f_x(x,y)\,\mathbf{i} + f_y(x,y)\,\mathbf{j}$

28. Tangent plane: $f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0) = 0$

29. Angle of inclination: $\cos\theta = \dfrac{|\mathbf{n} \bullet \mathbf{k}|}{\|\mathbf{n}\|}$

30. The Second Derivative Test: Suppose the second partials of f are continuous on an open region containing (a,b) and $\nabla f(x,y) = \mathbf{0}$ (i.e. (a,b) is a critical point). Let $d = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a) If $d > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a relative minimum.

b) If $d > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a relative maximum.

c) If $d < 0$, then $f(a,b)$ is neither a relative minimum nor a relative maximum.

d) The test is inconclusive if $d = 0$.