Homework Section 12.4

1. Let $\mathbf{u} = \langle -2, 5, -7 \rangle$ and $\mathbf{v} = \langle 2, -3, -6 \rangle$. Find the dot product, $\mathbf{u} \bullet \mathbf{v}$.

2. Repeat number 1 for $\mathbf{u} = -\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

3. Find $\mathbf{u} \bullet \mathbf{v}$, given $|\mathbf{u}| = 5$, $|\mathbf{v}| = 11$ and the angle between **u** and **v** is $\pi/3$.

4. Find the angle between $\mathbf{u} = \langle -2, 5, -7 \rangle$ and $\mathbf{v} = \langle 2, -3, -6 \rangle$. Give an exact expression and also an approximation to the nearest tenth of a degree.

5. Repeat number 4 for the vectors $\mathbf{u} = -\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

6. Decide if the given pairs of vectors are parallel, orthogonal or neither.

   a) $\mathbf{u} = \langle -2, 5, 2 \rangle$, $\mathbf{v} = \langle 0, -4, 10 \rangle$

   b) $\mathbf{u} = -\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

   c) $\mathbf{u} = -3\mathbf{i} - 2\mathbf{j} + \frac{3}{2}\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

7. Find the values of x that make the vectors $\langle 2, x, x \rangle$ and $\langle x, x, -3 \rangle$ orthogonal.

8. Find the vector projection of **u** onto **v**:

   a) $\mathbf{u} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$

   b) $\mathbf{u} = \langle -2, 5, -7 \rangle$ and $\mathbf{v} = \langle 2, -3, -6 \rangle$.

9. Find the work done in moving a particle from $P(3, -1, 0)$ to $Q(2, 3, 1)$ if the magnitude and direction of the force is given by $\mathbf{F} = \langle 5, 6, -2 \rangle$. Assume the magnitude of the force is in pounds and the magnitude of $\overrightarrow{PQ}$ is in feet (**Hint**: find the distance vector, $\overrightarrow{PQ}$, in component form first).

10. If $\|\mathbf{F}\|$ is 40 N, $\left\|\overrightarrow{PQ}\right\|$ = 3 m, and $\theta = 60°$, find the work done by **F** in acting from P to Q

11. The element in the ith row and jth column of a matrix A is denoted a_{ij}. Let $A = \begin{bmatrix} 2 & -3 & 8 \\ -5 & 1 & 0 \end{bmatrix}$.

    Which entry is a_{13}? Which entry is a_{22}?

12. The **determinant** of a 2×2 matrix $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by $\boxed{\det(A)=ad-bc}$.

Let $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$.

a) Find $\det(A)$

b) Find $\det(B)$

13. Let $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. To any entry, a_{ij}, in the matrix, there corresponds a 2×2 sub-matrix obtained by deleting the ith row and jth column of *A*. The following table gives the sub-matrices for entries in the first row of *A*:

Entry	Corresponding Sub-matrix
a_{11}	$\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$
a_{12}	$\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$
a_{13}	$\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

The **determinant** of a 3×3 matrix can be defined using the above sub-matrices as in the following formula:

$$\boxed{\det(A)=a_{11}\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}-a_{12}\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}+a_{13}\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}}$$

Find the following determinants using the above formula:

a) $\begin{bmatrix} -1 & 5 & 2 \\ 0 & 2 & 4 \\ 3 & -5 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 1 & 0 \\ -4 & 3 & 8 \\ -3 & 1 & 1 \end{bmatrix}$