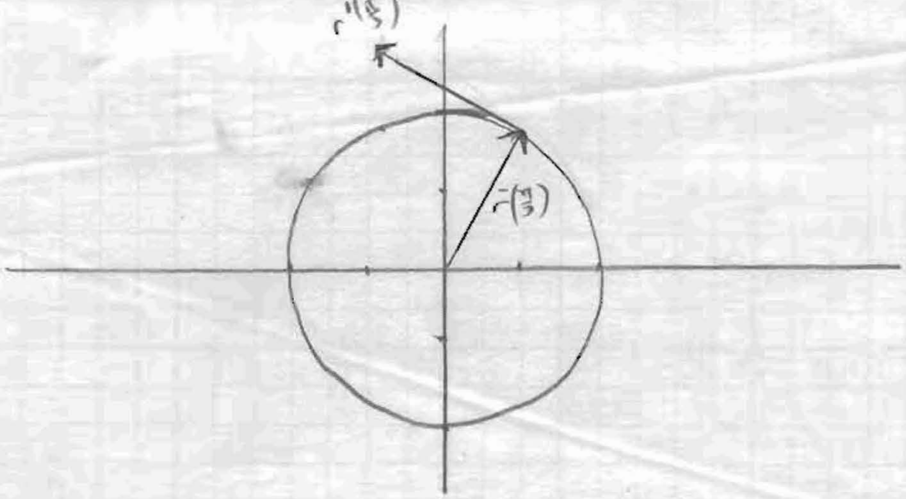
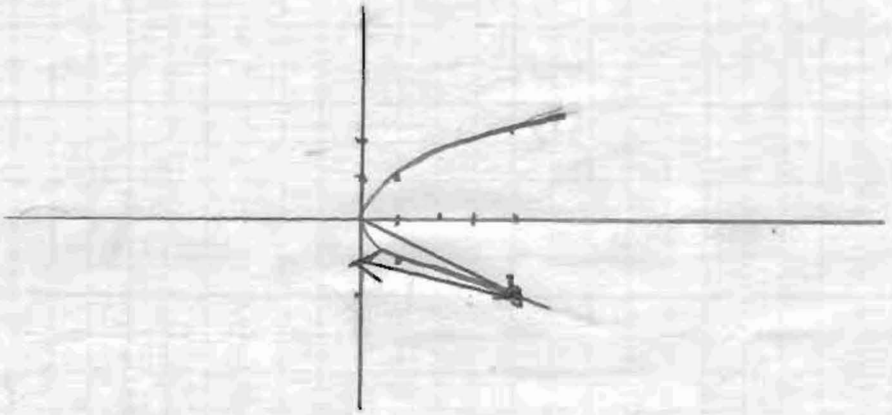
13.2

① a) $\vec{r}(t) = \langle 2\cos t, 2\sin t \rangle \Rightarrow r\left(\frac{\pi}{3}\right) = \langle 1, \sqrt{3} \rangle$

$r'(t) = \langle -2\sin t, 2\cos t \rangle \Rightarrow \vec{r}\,'\left(\frac{\pi}{3}\right) = \langle -\sqrt{3}, 1 \rangle$



b) $\vec{r}(t) = t^2\,\vec{i} + t\,\vec{j}, \quad t = -2$



$r'(t) = 2t\,\vec{i} + \vec{j}$

$r'(-2) = -4\,\vec{i} + \vec{j}$

$r(-2) = 4\,\vec{i} - 2\,\vec{j}$

13.2

2 a) $\vec{r}\,'(t) = 2\vec{i} - \vec{j} - 3\vec{k}$

b) $\vec{r}\,'(t) = \langle 2t, -\frac{5}{t^2}, \frac{1}{2}(2t-1)^{-\frac{1}{2}} \cdot 2 \rangle$

$= \langle 2t, \frac{-5}{t^2}, \frac{1}{\sqrt{2t-1}} \rangle$

c) $\vec{r}\,'(t) = 2\vec{i} + 3e^{3t}\vec{j} + 2\cos t \sin t\, \vec{k}$

3 $\vec{r}\,'(t) = 2\vec{i} - \vec{j} + \frac{3}{2\sqrt{t}}\vec{k}$

a)

$T(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} \Rightarrow T(4) = \frac{\vec{r}\,'(4)}{|\vec{r}\,'(4)|}$

$\vec{r}\,'(4) = 2\vec{i} - \vec{j} + \frac{3}{4}\vec{k}$

$|\vec{r}\,'(4)| = \sqrt{4 + 1 + \frac{9}{16}}$

$= \frac{\sqrt{89}}{4}$

$\boxed{\vec{T}(4) = \frac{4}{\sqrt{89}} \left(2\vec{i} - \vec{j} + \frac{3}{4}\vec{k}\right)}$

b) $\vec{r}\,'(t) = \langle 1, 2\cos t, -2\sin t \rangle$

$\vec{r}\,'\left(\frac{\pi}{6}\right) = \langle 1, \sqrt{3}, -1 \rangle$

$\left|\vec{r}\,'\left(\frac{\pi}{6}\right)\right| = \sqrt{1+3+1} = \sqrt{5}$

$\boxed{\vec{T}\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{5}} \langle 1, \sqrt{3}, -1 \rangle}$

(4) $\vec{r}(t) = 2t\,\vec{i} + (1-t)\,\vec{j} - 3\sqrt{t}\,\vec{k}$

$(8, -3, -6)$

$t=4$ produces this point

$\vec{r}\,'(t) = 2\vec{i} - \vec{j} - \frac{3}{2\sqrt{t}}\vec{k}$

$\vec{r}\,(4) = 2\vec{i} - \vec{j} - \frac{3}{4}\vec{k}$

$\boxed{x = 8 + 2t,\ y = -3 - t,\ z = -6 - \frac{3}{4}t}$

(5) a) $\vec{r}(t) = 2t\,\vec{i} + e^{3t}\,\vec{j} - \cos^2 t\,\vec{k}$

$\vec{r}\,'(t) = 2\vec{i} + 3e^{3t}\,\vec{j} + 2\cos t \sin t\,\vec{k} \neq \vec{0}$

so $\vec{r}$ is smooth (each component fct is continuous $\forall t$ and $\vec{r}\,'(t) \neq \vec{0}$)

b) $\vec{r}(t) = \langle t^2 + 5t^3, t^4 \rangle$

$\vec{r}\,(t) = \langle 2t, 15t^2, 4t^3 \rangle$

$\vec{r}$ not smooth at $t=0$

$\boxed{(-\infty, 0) \cup (0, \infty)}$

c) $\vec{r}(t) = \langle t^2, \frac{5}{t}, \sqrt{2t-1} \rangle$

$\vec{r}\,'(t) = \langle 2t, -\frac{5}{t^2}, \frac{1}{\sqrt{2t-1}} \rangle \neq \vec{0}$ for any t

not smooth unless $t \geq \frac{1}{2}$

$\boxed{\cancel{[\frac{1}{2}, \infty)}}$

Smooth on $\longrightarrow (\frac{1}{2}, \infty)$

13.2

(6)

a) $\int_0^2 (9t^2\vec{i} + 4t\vec{j} + 15t^4\vec{k})\,dt$

$\left[3t^3\vec{i} + 2t^2\vec{j} + 3t^5\vec{k}\right]_0^2$

$24\vec{i} + 8\vec{j} + 96\vec{k}$

b) $\int (2t\vec{i} + e^{3t}\vec{j} - \sin t\cos^2 t\,\vec{k})\,dt$

$t^2\vec{i} + \frac{1}{3}e^{3t}\vec{j} - \left[\int (\sin t\cos^2 t\,dt\right] k + \vec{C}$

aside

$\int \sin t\cos^2 t\,dt$

$u = \cos t$

$du = -\sin t\,dt$

$\frac{\cos^3 t}{3}$

$t^2\vec{i} + \frac{1}{3}e^{3t}\vec{j} + \frac{\cos^3 t}{3}\vec{k} + \vec{C}$

(7) $\vec{r}'(t) = 3t^2\vec{i} + 4t\vec{j} + 5t^4\vec{k}$, $\vec{r}(0) = \vec{k}$

$$\vec{r}(t) = \int \left(3t^2\vec{i} + 4t\vec{j} + 5t^4\vec{k}\right)dt$$

$$= t^3\vec{i} + 2t^2\vec{j} + t^5\vec{k} + \vec{C}$$

$$\vec{r}(0) = \vec{C} = 1\vec{k}$$

$$\vec{r}(t) = t^3\vec{i} + 2t^2\vec{j} + (t^5+1)\vec{k}$$