Homework Section 13.3

1. Find the exact value of the length of the curve given by $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}$, $-2 \le t \le 8$.

2. Set up the integral that gives the length of the following curve and then approximate that length, accurate to the nearest thousandth, using a calculator: $\mathbf{r}(t) = 2t\mathbf{i} + e^t\mathbf{j} - 3t^2\mathbf{k}$, $-5 \le t \le 6$.

3. Find the arc length function (starting from the point corresponding to $t = 0$) and use it to reparameterize $\mathbf{r}(t)$ in terms of arc length.

   a) $\mathbf{r}(t) = 5t\mathbf{i} + \cos t\mathbf{j} + \sin t\mathbf{k}$ b) $\mathbf{r}(t) = \langle 3t, 1+2t, 4-t \rangle$

4. Find the unit tangent and unit normal vectors (the unit normal vector is defined as $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$):

   a) $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}$ b) $\mathbf{r}(t) = e^{-t}\mathbf{i} + e^t\mathbf{j} - \sqrt{2}t\mathbf{k}$

5. Find the curvature:

   a) $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}$ b) $\mathbf{r}(t) = e^{-t}\mathbf{i} + e^t\mathbf{j} - \sqrt{2}t\mathbf{k}$

6. Find the curvature: $\mathbf{r}(t) = 2t\mathbf{i} + (1-t^2)\mathbf{j} - 3t\mathbf{k}$.

7. Find the curvature of $\mathbf{r}(t) = 2t\mathbf{i} + (1-t^2)\mathbf{j} - 3t\mathbf{k}$ at the point $(2, 0, -3)$.

8. Find the curvature of $y = 8x^4 - x$.

9. a) Where does the below curve have greater curvature, point A or point B? Justify your answer.

   b) Sketch the Circle of Curvature at point A and use its radius to approximate the curvature of the given curve at point A. Use millimeters as your units.