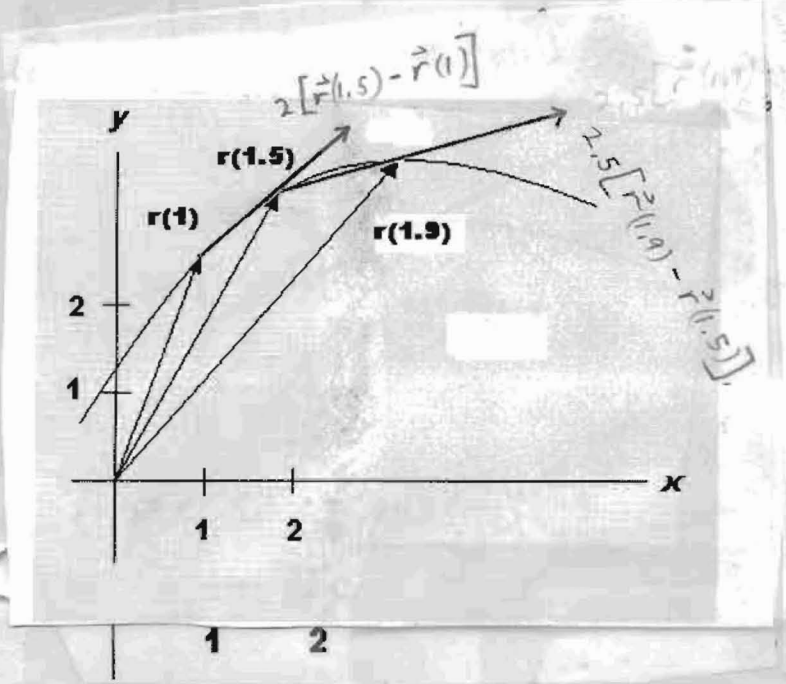
13.4

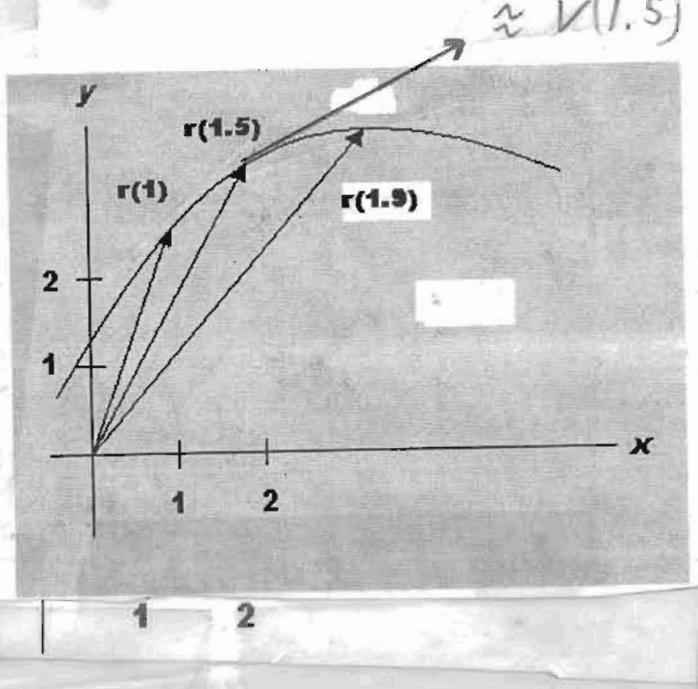
① a) Ave velocity $= \dfrac{\vec{r}(1.5) - \vec{r}(1)}{1.5 - 1} = 2\left[\vec{r}(1.5) - \vec{r}(1)\right]$

b) Ave velocity $= \dfrac{\vec{r}(1.9) - \vec{r}(1.5)}{1.9 - 1.5} = 2.5\left[\vec{r}(1.9) - \vec{r}(1.5)\right]$

c) $\vec{v}(1.5) = \lim_{h \to 0} \dfrac{\vec{r}(1.5+h) - \vec{r}(1.5)}{h}$

d) Average the lengths (according to the scale in the picture) of vectors from parts (a) and (b) to get ≈ 2.75 $\frac{\text{units}}{\text{time}}$ for the length of the tangent vector.





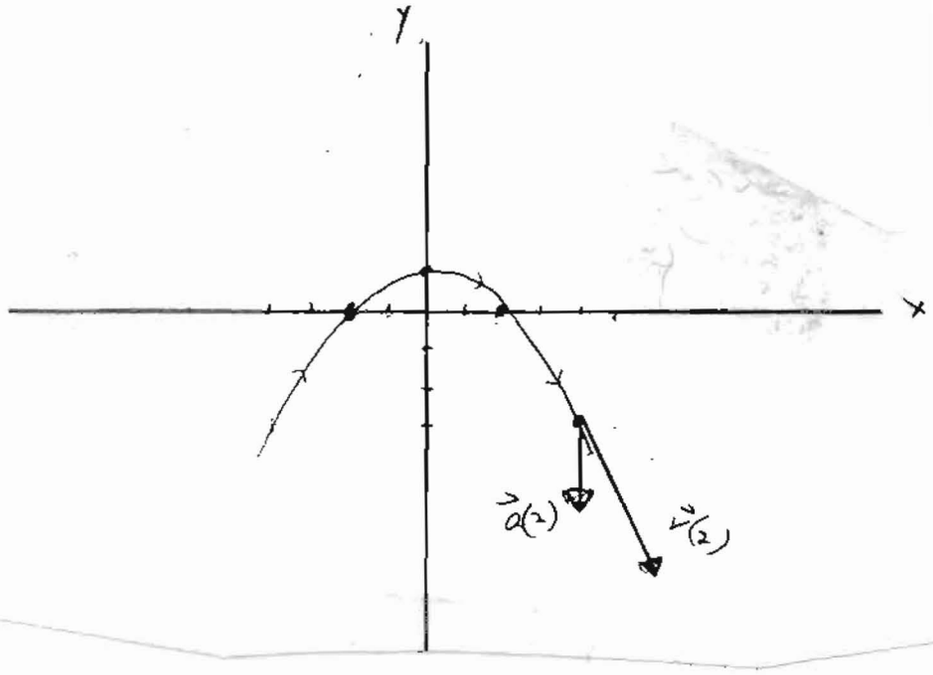
13.4

(2) a) $\vec{v}(t) = 2\vec{i} - 2t\vec{j}$

$\vec{a}(t) = -2\vec{j}$

speed $= \|\vec{v}(t)\| = 2\sqrt{1+t^2}$

b)



(3) a) $\vec{r}(t) = \langle 6t, 2, -3t^2 \rangle$

$\vec{a}(t) = \langle 6, 0, -6t \rangle$

$\|\vec{v}(t)\| = \sqrt{36t^2+4+9t^4}$

b) $\vec{r}(t) = 2\vec{i} - \sin t\,\vec{j} - \cos t\,\vec{k}$

$\vec{a}(t) = -\cos t\,\vec{j} + \sin t\,\vec{k}$

$\|\vec{v}(t)\| = \sqrt{5}$

(4) $\vec{r}(t) = t\vec{i} + \left(\frac{t^2}{2}+1\right)\vec{j} + (t-1)\vec{k}$

(5) $\frac{2}{25}$ sec

In numbers 6, 7, and 8 you may assume h=0.

(6) a) ≈ 14139.19 meters

b) ≈ 6122.45 meters

c) 400 m/sec

(7) ≈ 26.19 m/sec

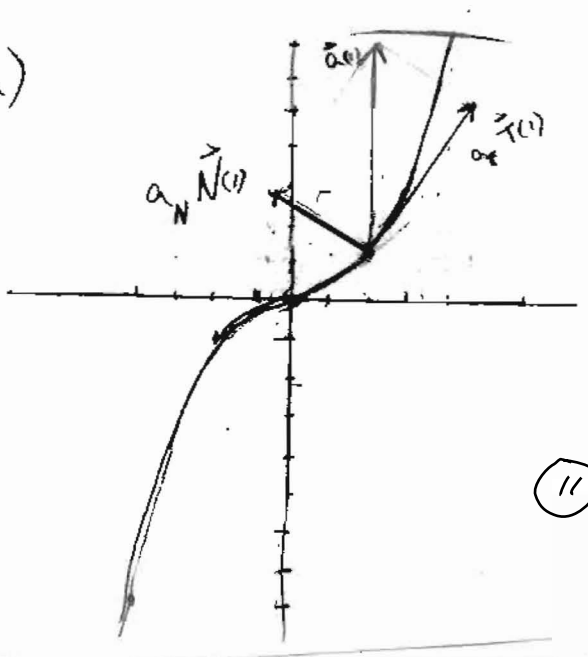
(8) $\approx 6.369°$ or $83.631°$

(9) a) $a_T = 0$, $a_N \neq 1$

b) $a_T = \frac{1}{e^{2t}+1}\left[-e^{-t}+e^{3t}\right]$

b) $a_N = \frac{\sqrt{2}}{e^{2t}+1}\left[1+e^{2t}\right]$

(10) a)



$a_T\vec{T}(1) = \frac{18}{13}(2\vec{i}+3\vec{j})$

$a_N\vec{N}(1) = \frac{12}{13}(-3\vec{i}+2\vec{j})$

(11) $\vec{T}(1) = \frac{1}{2}(\vec{i} - \sqrt{2}\,\vec{j} - \vec{k})$

$\vec{N}(1) = \frac{\sqrt{2}}{2}(\vec{i}+\vec{k})$

$\vec{B}(1) = -\frac{1}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j} + \frac{1}{2}\vec{k}$